The research topic
Non-Clifford gate on Gottesman-Kitaev-Preskill encoded optical qubits with nonlinear feedforward

The research activity

Background
With the currently widespread classical computers approaching their limits, there are growing expectations for the realization of quantum computers, which are expected to surpass classical computers in terms of computational speed. In order to perform practical quantum computation with fault tolerance, we have to construct error-correctable logical qubits. In a discrete variable system, we need a lot of qubits and large entanglements to construct logical qubits. In a continuous variable (CV) system, such as quantum optics dealing with quadrature values, we can use the degrees of freedom of wavefunction and construct a logical qubit in a single physical mode. Among such methods, the most promising candidate is Gottesman-Kitaev-Preskill (GKP) qubits [1]. The wavefunction of the GKP state has series of delta functions like a comb. The error can be detected as the shift of the peak position, and by pushing it back to the original position, we can execute error correction.

Quantum operations on qubits can be classified into Clifford gates and non-Clifford gates, and both types of gates are required for universal quantum computation. Clifford gates on GKP qubits correspond to Gaussian operations in the CV system, which are already realized in experiments dealing with propagating optical fields. On the other hand, non-Clifford gates on GKP qubits correspond to non-Gaussian operations, which are not realized because of experimental difficulties.

To realize the non-Gaussian operation, a method called gate teleportation can be used. In this method, we use a non-Gaussian state as an ancillary state. By using measurement and feedforward, we can indirectly apply the non-Gaussian operation on the input state. In the original GKP’s paper [1], two methods are suggested to implement the T gate, which is one type of the non-Clifford gates. One method is to use a cubic phase gate, which is a non-Gaussian operation and one of the CV universal gates. Cubic phase gate can be realized by CV gate teleportation [2]. The critical technology is nonlinear feedforward, which has been experimentally demonstrated in our laboratory. In recent research, however, it is pointed out that the cubic phase gate is not the most suitable for the T gate [3]. We found that the performance as the T gate can be improved by optimizing the gain of the cubic phase gate. However, the second method is more promising.

The second method is based on the magic state injection method. This is discrete variable gate teleportation. The non-Gaussian ancillary state is the logical T state, which can be distilled by magic state distillation protocol. We can realize this method in the physical CV system by simply implementing each logical part with its CV counterpart. However, the circuit constructed in this way requires four ancillary states. This means low performance as the T gate because we cannot neglect intrinsic noise from ancillary states.

Result
We proposed an experimentally feasible implementation of the T gate based on the magic state injection method (Fig. 1). Our proposed setup has a simple architecture with only two ancillary states thanks to using a beam splitter, a basic optical component, as an interaction gate. Our setup of the T gate has many similarities with
that of the cubic phase gate. In particular, the feedforward is the nonlinear feedforward, and the technology of the cubic phase gate can be directly applied. Therefore, by using the T state as the ancillary state instead of a cubic phase state, this setup can be readily realized. We numerically evaluate the performance of the T gate and revealed that our circuit works as an almost ideal T gate, while the performances based on the cubic phase gate approaches saturate. Moreover, our proposed T gate has better performance than the original circuit based on the magic state injection method because the intrinsic noise from ancillary states is suppressed thanks to the reduction of the number of the ancillary states.

By preparing the input and ancillary GKP states, we can realize the whole setup of our proposal. Therefore, our result is a crucial step toward the realization of fault-tolerant universal quantum computation with propagating optical fields.

![Implementation of the T gate](image)

**Features of our proposal**
- Based on magic state injection
- Only two ancillary states → High fidelity ~ 1 (less quantum duty)
- Nonlinear feedforward (demonstrated in e.g. [1])

**Fault tolerant universal quantum computation**

Fig. 1 Our proposal for experimentally feasible T gate and numerical evaluation.

³ The presentation at CLEO/EUROPE-EQEC 2021
I presented this result at CLEO/EUROPE-EQEC 2021, one of the largest international conferences on optical science in Europe, and had an active discussion. Two particularly critical comments were as follows:
I. How much is the threshold of the T gate on GKP qubits for the fault-tolerant quantum computation?
II. How much has the performance of the T gate been quantitatively improved by reducing the number of ancillary states from four to two?

I do not have clear answers to these questions now. I will try to clarify them in further research in the future. This presentation was highly evaluated and was selected as an OSA Foundation Best Student Presentation Grants.

References