（別紙様式５）

変革を駆動する先端物理・数学プログラム（FoPM）

国外連携機関長期研修　報告書

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Backgrounds

I visited Professor Yoh Tanimoto at Rome Tor Vergata University. The university is a historical place where many famous researchers on Algebraic Quantum Field Theory (AQFT) gather. AQFT is one of mathematical formulations of Quantum Field Theory (QFT), which is a standard framework to describe quantum mechanics and special relativity in a consistent way. QFT is natural in a physical sense, but it has been undefined mathematically in a form that satisfies physicists. AQFT is a certain mathematical description of QFT developed since 70's, and it focuses on the property of locality that the local observables possess, heavily utilizing the theory of operator algebras. However, from the birth of AQFT, its development has been independent from the community of high energy physics theory, and there is a lack of reflection of recent development of each other. One of the most prominent examples is the development of the concept of non-invertible symmetries on 1+1 dimensions in theoretical physics, which has been one of current trends since 2017. In AQFT, however, such a structure was already found at 70's. Mathematically, the concept is described in terms of fusion category. In physics, it is natural to consider not only bosonic systems but also fermionic systems. Fusin category describes non-invertible symmetry of bosonic systems. In fermionic systems, it is expected that we have superfusion category, which is a kind of extension of fusion category by $\boldsymbol{Z}_2$ grading. Then, since there is no AQFT-description of non-invertible symmetry of fermionic systems, I have been working on treatment of superfusion category in terms of operator algebras. This is my research theme.

Research

Describing my research theme more concretely, I am trying to build a theory of classification of superfusion categories that possess a structure of groups in certain sense. In usual fusion category, it was usual to consider the case where the isomorphism classes of simple objects form a group under the tensor product. Such fusion categories are said to be pointed. However, if we consider superfusion category, since the division superalgebra is not unique, we have several types of simple objects. Then, the situation becomes subtle when we want to equip a group structure on the isomorphism classes of simple objects. I had several discussions with Professor Tanimoto and Professor Sebastiano Carpi during the stay. In the discussions, I was requested to present a concrete example of superfusion categories satisfying the conditions. Thus, in the first half of the stay, I was working on construction of an example. However, I came to realize that in order to construct an example, I had to understand the classification of such superfusion categories in more detail in advance. For that, I had to study on a theory of central simple superalgebras and the categories of their modules. I noticed that central simple superalgebras form some higher categorical structures, but I could not find any literature describing such structure in detail. In order to make progress in my research, it is necessary to completely understand the structure. Thus, in the last half of the stay, I was mainly working on understanding the structure. I got to understand to some extent and the remaining problems became clearer.

Future direction

If I could build a theory of classification of superfusion categories with a group structure, it should be able to be derived from the representation category of a superconformal net. Professor Carpi is an expert on this area, thus I would like to have a discussion with him after I got an understanding of the classification. Also, I am interested in planar algebraic formulation of fusion category and its subfactor realization, so I will generalize the theory in case of superfusion category.

Daily life

My apartment was in Frascati, a small city about 30 minutes away from Rome by train. On weekdays, I commuted to university by bus. The last bus in the morning was at 9:30 and the last one in the evening was at 19:00, thus I could live a regular life. I took breakfast and lunch at a bar in the university. Sometimes I took lunch with professors and had conversation ranging from research to casual chat. The most memorable chat was on the distance between pillars of Italian bridges. After I returned to my apartment, I usually boiled spaghetti and eggs. On weekends, I sometimes went to Rome and saw something famous, such as the Vatican museums. However, since I did not like crowds, mostly I was walking along a mountain path in a nearby mountain. Since Frascati is a place where old Italian nobles build many big mansions, the path was very fun to walk. There were long old walls covered with moss, many trees, some of them fallen, the sound of birdsong, the sound of church bells, nice breeze, and sunlight through the trees, and sometimes a small river. There were a bench and a table along the path. There, I was reading Goethe while drinking Coca Cola.



A path in a mountain.