## 変革を駆動する先端物理・数学プログラム (FoPM)

## 国外連携機関長期研修 報告書

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I stayed in Brussels and visited Prof. Kenny De Commer at Vrij Universiteit Brussel. Main activities during this stay are some discussions with him and giving a talk at the seminar "Seminar on Quantum Groups, Hopf algebras and monoidal categories", which is organized by KU Leuven, ULB and VUB. In addition, I also participated in the conference "Higher structures in non-commutative geometry and quantum algebras", which was held at University of Lille from October 8<sup>th</sup> to October 11<sup>th</sup>.

Mainly I discussed with Prof. De Commer on the quantization of SL(2, R) and the extended SU(1, 1). The former is a mathematical object constructed and investigated by him in the paper published in 2024. An interesting point here is that his quantization of SL(2, R) is not a quantum group, just a coideal of quantum SL(2, C) which is the Drinfeld double of quantum SU(2). Hence we do not have a natural definition of the tensor product of representations of SL(2, R) arising from the definition of quantum SL(2,R). But recently he overcame this difficulty and found a tensor category which is equivalent to the category of representations of SL(2, R). Moreover he also realized that the technology developed in this work can be applied to a proof of the conjectural classification of irreducible representations of the extended SU(1, 1). During my stay, we started the discussion on this point. His first intuition failed to come to fruition, but we are keeping discussion via email.

I also had a progress on my own research project. I am mainly interested in the operator algebraic aspect of quantum groups and tensor categories. In my recent work, I focused on the deformation quantization of partial flag manifolds. In Donin's work, he classified all Poisson structures on such manifolds which are compatible with the action of compact Lie groups. Moreover he also shown that such Poisson structures can be quantized so that resulting quantization of partial flag manifolds has an action of the Drinfeld-Jimbo deformation of the compact Lie groups. What I am studying now is an analogy of this result in theory of operator algebras. Starting with Woronowicz's discovery of compact quantum groups, theory of quantum groups and their actions on operator algebras is one of main topic in theory of operator algebras. But there is no precise treatment of the notion of quantization, like the deformation quantization. In the paper I submitted in this summer, I give a construction which is applicable to obtain both of operator algebraic quantizations and deformation quantizations of partial flag manifolds. Then the next problem is to prove that these quantizations exhaust all operator algebraic quantizations of partial flag manifolds. This is a hard problem to solve in general. But, during my stay, I found that the answer is yes at least for SU(3). The basic idea is a parallel wih the work due to De Commer and Yamashita, which shows that actions of quantum SU(2) are classified by structured graphs. Their work depends on the universal property of the representation category of quantum SU(2). In my setting, I need the universal property of the representation category of quantum SU(3), which was already established by Kuperberg. As a consequence of the universal property, I obtained a joint equation with infinitely many variables, and surprisingly, solved it completely. A surprising point of this work is that this classification actually recovers the set of equivariant Poisson structures on SU(3)/T with 0-dimensional symplectic leaves. If we consider the family of such quantizations indexed by real numbers, presenting the Plank constant, the situation is similar to that of deformation quantization. But our result holds for each real number.

I have experienced various things during my stay at Brussel. I used to go shopping to Lidl, which is a supermarket near my home. There are various kinds of vegetables, cheeses and meats which cannot be found in Japanese supermarket. It was also really interesting that there are many bottles of beer. In Japan, Belgium is famous for chocolatiers like GODIVA, Neuhues, Leonidas and so on. But it is also famous for beer, and actually there are enormous kinds of beer in any supermarket as you can see in the picture below. I tried almost all beers in this picture. Their colors vary from light yellow to black, and their tastes also vary from acid to sweet like chocolate. I loved them and often went to a bottle shop "Beer Mania" located at Ixelles. I also visited some breweries, including 3 Fontainen, Oud Beersel, De Struise brewers and Westvleteren.

At the end of this report, I would like to express my gratitude to Kenny and staffs of Vrij Universiteit Brussel. They kindly supported me. I am also grateful to my homemates coming from various countries, and the students at University of Lille who brought me to Westvleteren.



Beers in a supermarket



From my room