（別紙様式５）

変革を駆動する先端物理・数学プログラム（FoPM）

国外連携機関長期研修　報告書

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Overview:

I visited Institut de Mathématiques de Bourgogne (IMB) in France and spent a few weeks there. I was invited to give a seminar on a previous work I did that discussed a new type of duality between quantum algebras and supersymmetric gauge theories of supergroup gauge theories. After giving a seminar, I spent the rest days discussing with Prof. Taro Kimura who is a specialist in my topic. We have been discussing the BPS/CFT correspondence of supersymmetric gauge theories which is my Ph.D. topic. Eventually, after this program was finished, we managed to complete a collaboration and we are still in contact with each other. I also met Yilu Shao who is a student of Prof. Kimura, and he is also my collaborator now.

Life in France:

I visited France for the first time in my life. It was a little chilly compared to Japan when I visited. I stayed in a place called Dijon, which is in the Bourgogne region, located east of Paris. France is like Japan in that there are people who speak English near the capital, but the further you go into the countryside, the less English is spoken. To get from Paris to Dijon, I took the TGV which is the French version of the Japanese bullet train (新幹線). The TGV seats were tighter than I expected, but the scenery outside was more spectacular than I imagined. During the TGV ride, perhaps because I was too nervous, an old woman smiled and spoke to me in French. Since I did not understand French, I could not communicate with her at all. It was the first time I regretted that I should have studied languages other than English more seriously in my undergraduate days. It seems that compared to Japan, greetings are considered relatively important in France. Even in the supermarket, before paying, it seems that it is natural to say “Bonjour!” even to the cashier. This was an interesting fact to me because in Japan I usually do not greet anyone.

After reaching Dijon, I spent most of my time at Bourgogne University where my host Prof. Kimura was giving a lecture. To go to the university, I rode a tram (路面電車) every day. I remember that the tram was crowded during commuting hours, though not as much as in Japan. On the weekends, I rode on this tram and spent time sightseeing in the countryside. It was an interesting experience.

Perhaps because Bourgogne University is a regional university, the term "physics" is referred to as experimental physics, while theoretical physics belongs to "mathematics”. The department I visited this time was what is called in Japan a "department of mathematics," where both rigorous mathematics and theoretical physics (particle theory, astrophysics, condensed matter physics) were all gathered together. The name of the department “Institut de Mathématiques” comes from this fact. Therefore, it seems to be normal to have discussions among different fields. There was a researcher in a neighboring laboratory who was mathematically studying random matrices, random tensor theory, and its application to

quantum gravity, and I also got to have lunch and discuss with him. I also learned that in France it is considered *cool* to study, understand, and do research related to mathematics. In fact, I saw some students talking about mathematics, probably something related to cohomology, while walking to school. This is probably related to the fact that the country has produced many great mathematicians, including Descartes, Fermat, Galois, and Poincaré.

Another characteristic I felt during my stay in France was the diversity of the various ethnic groups. There were people of all races on campus, including African, Asian, European, and American. Even at meals, consideration was given to whether the person eating the meal was vegan, etc. The food itself was also quite good. I learned that French bread is *really* hard. It was also my first time to eat Moroccan couscous and French escargot.

Research Activities:

My research topic for my Ph.D. is to understand the BPS/CFT correspondence of supersymmetric (susy) gauge theories using infinite-dimensional quantum algebras. As is well known, quantum field theory (QFT) is a theory to understand the interaction of fundamental particles appearing in nature. Usually, understanding quantum effects is difficult because perturbation is just an approximation. However, when we have supersymmetry, such kind of quantum effects are relatively controlled, and we can have a full understanding of the non-perturbative physical quantities. Instanton partition functions of susy gauge theories are examples of physical quantities that we can explicitly evaluate. An interesting property of these physical quantities is that they have rich underlying mathematical structures behind them and one of them is the quantum algebraic structure. In particular, we have a duality between partition functions of susy gauge theories and quantum algebras and this is called the BPS/CFT correspondence.

My goal for this stay was to get an understanding of the concept of "qq-characters". The qq-characters were first introduced by Nekrasov as an operator invariant under the action of adding and removing instantons. Usually, in physics, it looks natural that we focus only on one topological sector. This is because infinitesimal deformations of the gauge field will not change the topological sector. However, considering such kinds of symmetries relating different topological sectors leads to the concept of qq-characters. After this discovery, Kimura-Pestun constructed an operator version of the qq-characters which was found to be related to deformed W-algebras (Kimura here is my host Prof. Kimura). These discoveries explain the origin of the BPS/CFT correspondence. Roughly speaking, the qq-characters are just an operator lift-up of the instanton partition functions.

During my stay, Prof. Kimura gave me 3 to 4 hours of lectures every day about what he has been doing for years and explained to me the basic concept of qq-characters. Usually, there are two ways to discuss the BPS/CFT correspondence from the algebraic perspective. One way is to start from a large algebra such as quantum toroidal algebras, consider their representations, and then obtain physical quantities. The representations appearing from these algebras physically correspond to the action of adding and removing the instantons. Another way is to use the qq-characters as explained in the previous paragraph. I already had experience and knew very well how to use the former way but did not have any experience with the latter way. This stay was very valuable because I managed to expand my understanding.

In particular, we discussed the gauge origami system, which is a complicated susy gauge system where intersecting gauge theories appear. Recent examples are the spiked instantons, tetrahedron instantons, and the magnificent four system. These systems appear as low energy limits of the world volume theories of D-branes of various dimensions wrapping the non-compact cycles of the $\mathbb{C}^4$ geometry. The quantum algebraic understanding of them was an unsolved problem and we managed to determine the algebraic structure. The discussion we did during my stay in France eventually led to a paper we wrote one year later after my trip in France (arXiv: 2310.08545).

Acknowledgements:

During the stay, I managed to learn many things not only topics related with my research but also what it is to live and do research abroad. Since Prof. Kimura is a Japanese and his hometown was the same with me, through chatting with him, I was able to get an idea of what it would be like to work as a researcher abroad. I was able to eliminate psychological resistance to conducting research abroad. Even though I made many mistakes during my stay in France, I feel that I was able to grow as a person. As a pet theory, one advantage of experiencing Ph.D courses is the experience of failure. The stay in France certainly enabled me to do that. Finally, I would like to express my gratitude to Prof. Kimura for hosting my stay, booking everything from flights, hotels, TGV, etc. I also appreciate Yilu who brought me and showed me around the university. I will never forget those nameless people who tried to communicate with me, although I could not speak French, and helped me when I was in trouble on the street.

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