(Form 5)

Forefront Physics and Mathematics Program to Drive Transformation (FoPM)

Report for the International Research Experience

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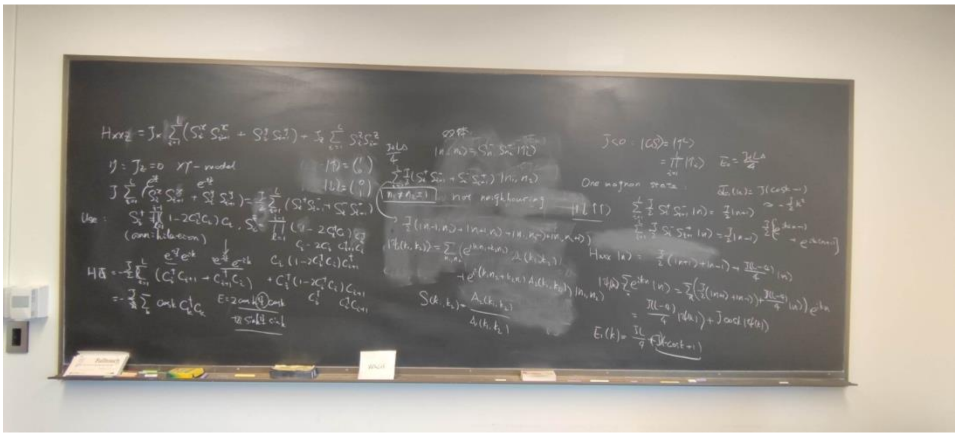
During this period of stay in Princeton University, I developed the Yang-Lee theory of entanglement transition of ground states for two integrable models to investigate the relationship between Yang-Lee zeros and entanglement transition.

Yang-Lee zeros, defined as the zero points of the partition function of the canonical ensemble, give rise to a mathematical explanation of the singularity of thermodynamic observables at phase transition points. In the classical ferromagnetic Ising model, Yang and Lee investigated the distribution of zeros of the partition function in the presence of an imaginary magnetic field to understand the origin of the nonanalyticity of the ferromagnetic phase transition when one increases the temperature. The thermal phase transition takes place when Yang-Lee zeros touch the positive real axis on the complex plane of fugacity. It was later extended to more systems beyond the Ising model to understand the properties of the phase transitions, especially the origin of the singularity in thermodynamic quantities in many-body systems for not only classical phase transitions but also quantum phase transitions. My previous published work is on how to utilize the Yang-Lee zeros to understand the origin of essential singularity in BCS superconductivity.

Despite the great success of the application of Yang-Lee zeros to explain the origin of singularity in thermodynamic quantities, its application to understanding entanglement transitions of ground states is still ambiguous to us. Entanglement, as one of the most striking phenomena in quantum physics, plays an essential role in understanding the correlation properties between different partitions in quantum systems. Given a ground state $|\psi\rangle$ and a bipartition of the space into A and $\bar{A}$ of a quantum system, we can define the reduced density matrix of A as $\rho_A = -Tr_{\bar{A}}[\rho]$and the entanglement entropy between A and $\bar{A}$ as $S = -Tr_A[\rho_A \log\rho_A]$ to quantitatively describe the entanglement of the ground state. There are three types of behavior of the entanglement entropy: area law, subarea law and volume law. Generally, there are two types of transitions in the many-body systems: volume-area law transition and subarea-area law transition. During this stay in Princeton University, I try to apply Yang-Lee zeros to understand the origin of discontinuous jump of entanglement entropy from subarea law to area law for the ground states of two integrable models. One is the free electron model and the other is interacting and non-perturbative fermionic model. I calculated the zeros of both models and found the distribution of both models, which can indicates the type of entanglement transition of ground states. The figure below shows the discussion with my friends.



In addition to this project, I also engaged in discussions with other groups on how to calculate the entanglement entropy in the gapped systems and argue how the entanglement entropy can be related to quantum geometry in general. To begin with, we consider an integer quantum Hall state which has an integer filling of the Landau levels. We calculated the entanglement entropy of the lowest Landau level and the second Landau level to see how the entanglement entropy scales with the size of the subsystem from the single-particle correlation function. Even though we know that the entanglement entropy should behave as area law in the gapped systems, we still have no idea whether the coefficient has anything to do with the quantum geometry such as the Fubini-Study metric and Berry curvature. Our results show that the coefficient of the entanglement entropy will increase with the Landau level, which indicates that the quantum metric plays an important role in physics here. Even though this result has been numerically calculated before, we reached the analytical results with some appropriate approximations for the first time. To further understand the relation, we define a new quantity: quantum geometric entanglement entropy and found that the leading term in the entanglement entropy is just proportional to an integral of the quantum metric. This may open a new gate to understanding how geometry gives rise to entanglement in many-body systems, especially fractional Chern insulators, which are of great interest to us.

To summarize, I really cherish this opportunity to communicate with the experts there on different aspects, including theoretical condensed matter physics, experimental and theoretical ultracold atomic physics, and also quantum information theory. In Shinsei's group, they all have their own directions so I can learn a lot from discussions with them. For example, my Switzerland friend Bastien mainly focuses on topological insulators while my Chinese friend Yuhan focuses on the multipartite entanglement entropy. In addition, I feel that my English speeching skills are greatly improved during discussions. I learned how to give talks and seminars in the group meetings. My horizons and interests were broadened.