（別紙様式５）

# 変革を駆動する先端物理・数学プログラム（FoPM）

## 国外連携機関長期研修　報告書

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I stayed the Center of Mathematical Sciences and Applications (CMSA) in Harvard University for one month. My host is Professor Dan Freed. Main activities are some discussions with Dan and Mayuko Yamashita, who stayed at same time in Boston, participation for some seminars and lectures and discussion with students there.

Cambridge in Massachusetts, where Harvard University is located and MIT is also present, had very good public safety, perhaps due to being a university town. Ignoring the fact that the cost of living is 2 to 3 times higher than in Japan, it was a comfortable and pleasant city to live in. Because Harvard University and MIT are so close to each other, students often interact and collaborate. They organize joint seminar and seem to be working hard to improve each other.

I attended various seminars and experienced firsthand how homotopy theory, higher category theory, and related physics are gaining momentum. This field is rapidly advancing in the United States, and other places, with theories evolving quickly. However, Japan is somewhat behind because there are very few researchers in this area. It was very stimulating to hear a variety of discussions that are not commonly heard in Japan during this visit. For instance, there were lectures on Freed's quantum theory, Hopkins' lectures on infinity categories, and lectures on topological modular forms (TMF) by a postdoc. Many seminars had participation from both Harvard University and MIT students. In the official topology seminar, various topics related to both topology and physics were covered, including Mayuko Yamashita's talk on the relationship between TMF and heterotic string theory, talks on the Stolz-Teichner conjecture, talks on fusion category, and talks on defects. There were also unofficial seminars organized by students, including seminars on homotopy theory in general and seminars on topological automorphic forms (TAF). Additionally, at CMSA, there was a weekly Q&A seminar titled "What is ○○?" where mathematicians and physicists gathered to have short talks while having lunch. It was an enjoyable initiative.

TMF is one of generalized cohomology theory that appears when reconstructing Witten's genus, originally constructed from a physical perspective, at the spectrum level. Elliptic cohomology is also one of generalized cohomology theory closely related to TMF. The global sections of a sheaf taking values in the $E_\infty$ ring spectrum of elliptic cohomology over the moduli stack of elliptic curves give rise to TMF. Elliptic cohomology and TMF have long been suggested to be related to 2-dimensional supersymmetric fields theory. In particular, since Stolz-Teichner proved an isomorphism with KO theory in the case of 1-dimensional field theory, the relationship between TMF and 2-dimensional supersymmetric field theory has been known as the Segal-Stolz-Teichner conjecture. The mathematical treatment of field theories is an ongoing challenge, so there is no mathematically rigorous formulation for this conjecture.

On the other hand, there is Freed-Hopkins' result that reflection positive invertible field theory is classified by the Anderson dual of the bordism group. Since the anomaly in field theory can be formulated as an invertible field theory, Freed-Hopkins conjecture allows us to handle anomalies using the Anderson dual of the bordism group.

Yamashita's talk translated physical problems into mathematical problems through the Segal-Stolz-Teichner conjecture and Freed-Hopkins conjecture. The aim was to prove that the anomaly of a certain theory vanishes. Furthermore, the talk showed that the map induced to the cofiber is related to the Anderson self-duality of TMF. Outside of the seminars, Yamashita engaged in discussions with Freed, visiting researchers, and students of Hopkins about topics related to these results and the Segal-Stolz-Teichner conjecture. I actively participated in these discussions.

The remaining time was spent interacting with students and postdocs. Most students working in fields closely aligned with my interests were usually students of Professor Hopkins. I particularly enjoyed discussions about the collaborative papers of these students. Most students working in fields closely related with my interests were usually students of Professor Hopkins. This was the most beneficial aspect for my research. I have been

conducting research with a focus on the relationship between algebraic topology and physics. My previous results, including the alternative proof for the bulk-edge correspondence and Bott periodicity, also relate directly to the connection between topological K-theory and condensed matter physics. As mentioned earlier, the Freed-Hopkins conjecture establishes the relationship between the Anderson dual of the bordism group and invertible field theory, while the Segal-Stolz-Teichner conjecture asserts the relationship between TMF and 2-dimensional supersymmetric field theory. The paper by students of Hopkins shows that the long exact sequence related to the Anderson dual of the bordism group is relevant to the defect anomaly matching condition in physics. Hence, this paper also shows the connection between algebraic topology and physics, which relates closely with my interests. Mathematically, there is a fiber sequence related to Thom spectra, and taking the bordism group reveals its connection to the Pontryagin-Thom construction. Furthermore, taking the Anderson dual of the bordism group, due to Freed-Hopkins' results, allows us to understand the connection to anomalies in field theory. It is argued that, due to exactness, if certain obstructions in field theory vanish, it localizes to defect theory. It was a significant gain to know that students are writing papers on such topics. Also, I discussed the foundations of chromatic homotopy theory with other students.

In the fourth week, I had the opportunity to give a talk at the topology seminar at Johns Hopkins University in Baltimore, Maryland. I also interacted with students there and received positive feedback from faculty members.