## 変革を駆動する先端物理・数学プログラム (FoPM)

## 国外連携機関長期研修 報告書

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日程	西暦 2023 年 9月 28日 ~ 西暦 2023 年 12月 23日

I report the summary of this visit and what I gained from it.

Throughout the visit, I always participated in weekly Logic Seminar and Complexity Seminar organized by Institute of Mathematics, Czech Academy of Sciences, at Žitná except for ones during my visit to Passau (as for this nested visit, see below). In most of working hours, I communicated with Ph.D. students of my host sharing an office at Křižíkova, where Department of Algebra was, did surveys on my projects, tried possible implementations of new ideas, and had face-to-face meetings with powerful researchers in Europe for discussions.

During October 29<sup>th</sup>-November 2<sup>nd</sup>, I visited Professor Moritz Müller at the University of Passau in Germany and discussed one of my main interests with the scholars there (it included a seminar talk); comparison between the strengths of the injective pigeonhole principle and the full-version of the bijective pigeonhole principle over the bounded arithmetic  $V^0$  as the base theory. It was also a great chance meeting with Professor Tobias Kaiser and his Ph.D. students there. Since they were specialists of o-minimal theory and real algebraic geometry, which I had been also interested in motivated by model theory of bounded arithmetics such as Shepherdson's theorem and subsequent works, I was able to learn many helpful literatures, preceding works which I should look at, and active communities (even some Japanese ones) in the field.

After the visit, on November 20<sup>th</sup>, at Logic Seminar at Žitná, I gave another seminar talk on my trial to unify the understanding of proof theoretic strengths of bounded arithmetics and the hierarchy of  $I\Sigma_k$ -fragments of PA in terms of a game notion allowing backtracking options.

In the late November and December, I also visited several professors at Žitná in order to discuss possible improvements of the game notion above and to present another collaboration with my colleagues at the University of Tokyo.

Three principal things I gained through the visit are as follows.

Technically, I have found potential of the game notion (roughly saying, we can formalize combinatorial games for which we can extract a winning strategy of one player from a given proof over one of the theories from Buss's hierarchy) I formalized for obtaining another proof of Ajtai's theorem, one of the strongest superpolynomial proof-size lower bounds for concrete propositional proof systems (the field has been seeking for proof techniques other than switching lemma for establishing further superpolynomial proof-size lower bounds, and another proof of Ajtai's theorem technically independent from switching lemma would be appreciated). It is still ongoing research, but partial results I obtained together with a Ph.D. student of my host will be comprehended and would be made public in the near future. On the other hand, I have come to recognize that the injPHP v.s.ontoPHP problem I tackled in my master's thesis is more difficult than I initially expected, and therefore is very interesting. The main difficulty is a lack of switching lemma for injective PHP-trees I formalized in the thesis. Despite several fresh ideas from experts were tried throughout this visit to obtain a counterpart, it is still unclear how to overcome the difficulty. I now believe that a solution to this problem would give a novel proof technique in the realm of Proof Complexity.

Not only progress of my own works, but also my colleagues' works were of course very important. This visit was a great opportunity for me to learn which subjects scholars on the very front line of Proof Complexity were tackling. The talks and teachings of experienced experts have been exceedingly helpful for me; the number of papers directly connected to Proof Complexity is, if we simply count those publicized via a common platform Electric Colloquium on Computational Complexity, roughly 2-3 papers per day nowadays, which makes practically impossible to read all papers thoroughly. Thus, it has been difficult for a student like me, studying in Japan, which is physically quite distant from the center of the area, to grasp a comprehended picture of the field, especially the current frontiers.

Secondly, I tried to socialize as many students at the same faculty as possible. Common affiliations were departments of Algebra, Computer Science, Numerical Analysis, Physics, and Philosophy. Since Proof Complexity, my major, is a field where Computational Complexity and Mathematical Logic meet, the open communications I had with those students were often inspiring for me. The experience also gave me an understanding of the current situation of academias in the Czech Republic and surrounding European countries.

I have not only kept in mind to absorb mathematical technicalities, but also learned how to become a good international host and educator. Professor Jan Krajíček was a truly sincere host, being considerate of colleagues having different cultural backgrounds. Furthermore, the seminar talks (including mine) were extremely instructive for me to learn which conventions or approaches should be taken to make our works more understandable worldwide. Also, along the communications with my colleagues, I was able to know standard curricula of Mathematical Logic and Computational Complexity for students of various levels at Charles University, which would be helpful in my future career.

