変革を駆動する先端物理・数学プログラム (FoPM)

国外連携機関長期研修 報告書

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I stayed in Manhattan, Kansas to talk with Victor Turchin at Kansas State University. In addition, I stayed in Eugene, Oregon (from Oct 22nd to Nov 2nd) to talk with Dev Sinha at University of Oregon. My main achievements during my stay are as follows.

(0) Revised my first paper.

Title: Cocycles of the space of long embeddings and BCR graphs with more than one loop, arXiv2212.01573 (1) S locities to the space of long embeddings and BCR graphs with more than one loop, arXiv2212.01573

(1) Submitted a new article to arXiv (Oct 17th).

Title: Two graph homologies and the space of long embeddings, arXiv.2310.10896

(2) Gave a talk at Topology Seminar at Kansas State University (Sep 1th)

Title: Non-trivial cycles of the spaces of long embeddings detected by 2-loop graphs

(3) Gave a talk at Topology/Geometry Seminar at University of Oregon (Oct 24th)

Title: Two graph homologies and the space of long embeddings

(4) Wrote a draft of my forthcoming paper

Title: Decorated graph complex and the space of long embeddings modulo immersions

The purpose of this stay was to learn and discuss an algebraic topological approach to the space of long embeddings. And I aimed to establish relationships between this approach and a geometrical approach I had researched before my stay. Overall, the plan went smoothly and I achieved my goal.

The article (1) is about this relationship. Two combinatorial objects to describe the homology of the space of long embeddings are known: One is the hairy graph homology, which was invented from the algebraic topological approach. The other is BCR graph homology, invented from the geometrical approach. In this paper I constructed a monomorphism from the hairy graph homology to BCR graph homology, though the latter graph homology is quite modified.

The talk (2) is based on my paper (0). Through this talk, I practiced to make a presentation on my research in English to researchers from all over the world. I received several questions from the audience about some notation (on graphs and configuration spaces, for example) I introduced. Those questions helped me improve my exposition of (0).

The talk (3) is based on the article (1). I received several questions on how to interpret my results. Thanks to these questions and discussions after my talk, I found further applications of my results of (0) and (1) (such as results for different components of homotopy groups of the space of long embeddings).

The paper (4) aims to apply the results of (3) to (0). To complete the proof of my main result in (4), I need to use a technique from algebraic topology such as spectral sequence and bar construction. I studied and learned these techniques during my stay.

I talked with Turchin every week. Through the discussions, I learned the algebraic topological approach and the hairy graph homology. The knowledge I got from him helped me write the introduction of the paper (1) and complete the proof of one of my main results of (4). Turchin also helped me revise the exposition of (0).

Sinha provided me with future problems on the space of long embeddings related to my research. And we exchanged our ideas on these problems.

During my stay, I communicated with many researchers, including students of Turchin and Sinha. Through discussions with them, I learned various approaches to studying the space of long embeddings that I hadn't known before. At the same time, I could also promote my research to them. I also attended weekly topology seminars at Kansas State University, and I caught up recent trends in topology.

I had valuable cultural experiences during my stay. I stayed in a 4-bedroom apartment in campus. I enjoyed communicating with residents from various countries. Manhattan, Kansas is a town in the countryside with few tourist spots. But I could enjoy local foods, museums and events that couldn't be experienced in big cities. Through these experiences, I learned about local culture in the United States.

Acknowledgment

I'm deeply grateful to my hosts Victor Turchin and Dev Sinha. Discussions with them highly motivated my research. I also thank Kansas State University and University of Oregon for their hospitality. This stay was supported by Forefront Physics and Mathematics Program to Drive Transformation (FoPM), a World-leading Innovative Graduate Study (WINGS) Program, the University of Tokyo. I appreciate the support.

Pictrue1: Kansas State University



Picture2: Topology Seminar

