（別紙様式５）

変革を駆動する先端物理・数学プログラム（FoPM）

国外連携機関長期研修　報告書

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Overview

From February to March 2023, I visited the California Institute of Technology (Caltech) and worked with Professor Xiao-Gang Wen, who is a Cecil and Ida Green Professor of Physics at the Massachusetts Institute of Technology and was at Caltech as a Moore Distinguished Scholar during my stay. We worked on the relation between finite symmetries of quantum many-body systems and topological orders in one higher dimension. Specifically, we figured out how to compute various link invariants of topological orders in 2+1 dimensions by using the symmetry operators, charge operators, and their composite for general finite group symmetries in 1+1 dimensions.

Research activity

In recent years, the generalized notion of symmetry has attracted much attention in high energy physics and condensed matter physics. From a traditional point of view, symmetry is defined as the group of transformations that leave the system unchanged. However, recent studies have revealed that the symmetry of a quantum many-body system should be defined as the algebraic structure of topological defects rather than the group of transformations. Remarkably, symmetry defined in such a way is not necessarily described by a group: finite symmetries in (d+1)-dimensional spacetime are expected to be described by fusion d-categories in general, which include finite groups as special cases. For example, finite symmetries in 1+1 dimensions are generally described by fusion categories and therefore are called fusion category symmetries.

Figure 1. The building of the physics department (left).

Finite symmetries of quantum many-body systems are closely related to topological orders in one higher dimension. Specifically, a (d+1)-dimensional quantum many-body system with a finite symmetry can be realized by an appropriate topological order on a (d+2)-dimensional slab. The topological order in the (d+2)-dimensional bulk is uniquely determined by the finite symmetry in (d+1) dimensions. This relation between finite symmetries in (d+1) dimensions and topological orders in (d+2) dimensions suggests that a (d+1)-dimensional quantum system with a finite symmetry somehow contains the data of the corresponding (d+2)-dimensional topological order. However, how to extract the data of the topological order from the quantum many-body system in one lower dimension is not obvious because topological orders in (d+2)-dimensions are (partly) characterized by the braiding of d-dimensional extended objects, whereas the braiding of d-dimensional objects cannot be defined in (d+1)-dimensional spacetime.

In order to understand how to extract the data of topological orders from finite symmetries in one lower dimension, I started to consider computing the braiding data such as the modular S- and T-matrices of (2+1)-dimensional topological orders from finite group symmetries in 1+1 dimensions. The (2+1)-dimensional topological orders corresponding to finite group symmetries in 1+1 dimensions are called the quantum double models. When the symmetry is abelian, it was already known that we can compute the modular S- and T-matrices of the quantum double model by using patch operators in 1+1 dimensions, which are the symmetry operators and charge operators defined on a patch of a one-dimensional space. However, a generalization of this computation to non-abelian symmetries was unclear due to several subtleties arising from the non-commutativity of the group multiplication law.

After discussions with Professor Wen and his student Arkya Chatterjee, we found explicit expressions of patch operators for a general finite group symmetry in 1+1 dimensions, which enable us to correctly compute the modular S- and T-matrices of the quantum double model in 2+1 dimensions. Remarkably, the computation based on the patch operators does not make use of gauge non-invariant quantities such as F-symbols and R-symbols of the braided fusion category that describes the topological order. Furthermore, the patch operators also make it possible for us to compute various link invariants beyond the modular data, e.g., the link invariant associated with the Borromean ring.

Future directions

The computation of various link invariants of the quantum double model is the simplest non-trivial demonstration of the relation between finite symmetries and topological orders in one higher dimension. There are several possible generalizations of this computation. For example, it would be interesting to incorporate anomalies of finite group symmetries in 1+1 dimensions. The (2+1)-dimensional topological order corresponding to an anomalous finite group symmetry is known as the twisted quantum double model. We should be able to compute various link invariants of the twisted quantum double model similarly if we find the correct patch operators for an anomalous finite group symmetry. More generally, the prescription to compute link invariants by using patch operators should apply to more general finite symmetries described by fusion categories. The topological order corresponding to a general fusion category symmetry is realized by the Levin-Wen model, whose anyon excitations are described by the Drinfeld center of the fusion category. Thus, patch operators for a general fusion category symmetry would provide a physical way to compute the Drinfeld center of a fusion category. Another interesting generalization is to consider patch operators for finite group symmetries in higher dimensions. These patch operators would enable us to compute the data of higher dimensional topological orders, which are much less understood than topological orders in 2+1 dimensions.