（別紙様式５）

変革を駆動する先端物理・数学プログラム（FoPM）
国外連携機関長期研修　報告書

氏　　名	奥山 義隆
所属部局	理学系研究科　　　　物理学専攻
受入先	International Centre for Theoretical Sciences, Tata Institute of Fundamental Research
日程	西暦 2022 年 1 月 9 日　～　西暦 2022 年　1 月 22 日

As the International Research Experience, I participated in the Kavli Asian Winter School on strings, particles, and cosmology from January 9, 2022, to January 22, 2022. This winter school aims to give opportunities for young researchers to take intensive lectures (three lectures and one tutorial for each topic this year) on recent developments in particle/astrophysics by leading scientists. And I found it very valuable to join this winter school to broaden my perspective and determine the direction of my future research.

This Winter School is usually held on a rotating basis by Asian countries. This year, this school was supposed to be held at the Tata Institute of Fundamental Research in India. Unfortunately, due to Covid-19, it took place online.

All lectures were very beneficial to me. In recent years of increasingly compounded research, it is valuable to have the opportunity to reconfirm what is known and assumed in the research area. Besides that, follow-up tutorials on the weekends were instructive to deepen my understanding of the lecture topics.

In what follows, I would like to comment on three topics that particularly impressed me.

- Tom Hartman's lecture "Entanglement in QFT and Quantum Gravity."

He went from the definition of entanglement entropy in quantum field theory to the recently proposed resolution of the black hole information paradox via the island formula of the entropy of Hawking radiation.

He started with a nice graphical description of path integral and demonstrated how to calculate Renyi entropy by considering n-copies of our universe. Despite the difficulty of the direct calculation, we can evaluate the entanglement entropy by performing proper manipulations on the Renyi entropy. After having exemplified the entanglement entropy in two-dimensional conformal field theories, he got into how to evaluate the entanglement entropy in the presence of gravity. Even though we assume the path integral of quantum gravity is defined with a fixed boundary condition on the asymptotic boundary, it is generally impossible to evaluate because of the fluctuations of the spacetime. However, with the help of semi-classical approximation, it turns out that we can estimate the gravitational path integral as a summing over saddle-point values of the effective action, among which the leading saddle plays the dominant role. The point is that, with that estimation, the entanglement entropy in the gravitating system takes the form of generalized entropy, namely, black hole entropy plus matter entropy outside the horizon.

In the last (third) lecture, he first took a quick review of the black hole information paradox. Then, he demonstrated that it is possible to solve the problem by considering the non-perturbative effect in the gravitational path integral. This non-perturbative effect arises from a non-trivial saddle point called the Replica wormhole, a sort of wormhole connecting multi-gravitating regions in the Replica calculation. Lastly, he closed his lecture posing further directions and remaining problems on this topic.

These lectures were very clear and instructive, which was good even for me to sharpen my understanding as I have written several papers on this topic.

- Simon Caron-huot's lecture "S-matrix and Conformal and Bootstrap."

In these lectures, he described his recent work on S-matrix bootstrap to restrict low-energy effective field theory data.

He first introduced three ingredients to constrain physical data: symmetry, positivity, and crossing. The guiding principle of "bootstrap" is to use these three constraints to curve out the theory space of consistent quantum field theories.

Because symmetry, positivity, and crossing are tied to the foundations of quantum field theory, it is generically hard to give intuitive explanations. His explanations, combining analogies with signal models and geometrical optics, were very intuitive and, at the same time, showed his profound knowledge of physics.

After going over the basics, he talked about the effective field theory (EFT) bootstrap. With the help of the analyticity of the 2-to-2 scattering amplitude, we can write down its dispersive sum rule. There, low-energy EFT coefficients are related to the high energy part of the imaginary part of the scattering amplitude integrated over the center of mass-energy. By expanding the sum rule order by order in terms of Mandelstam variables and using the positivity of partial wave coefficients, we can derive infinitely many constraints on the EFT data. There is an efficient way to solve the infinite number of restrictions from a numerical perspective called SDPB, an open-source solver. He demonstrated how to solve the constraints via SDPB.

According to his talk, what he improved in his recent paper is the boundedness of the scattering amplitude in the ultra-high energy limit. It is necessary to assume the boundedness in contour deformation on the Mandelstam plain to derive the dispersion relation. But, the boundedness breaks down once we incorporate the loop correction. However, he has found a way to overcome this difficulty by smearing scattering amplitude over transverse momenta.

I did know the philosophy of S-matrix bootstrap, but recent progress in the field is so quick and involved that I cannot catch up. However, without getting into the technical details, he nicely told us the basics of S-matrix bootstrap enough to understand his recent studies.

- Dam Thanh Son's lecture "QFT/CFT in Condensed Matter Physics."

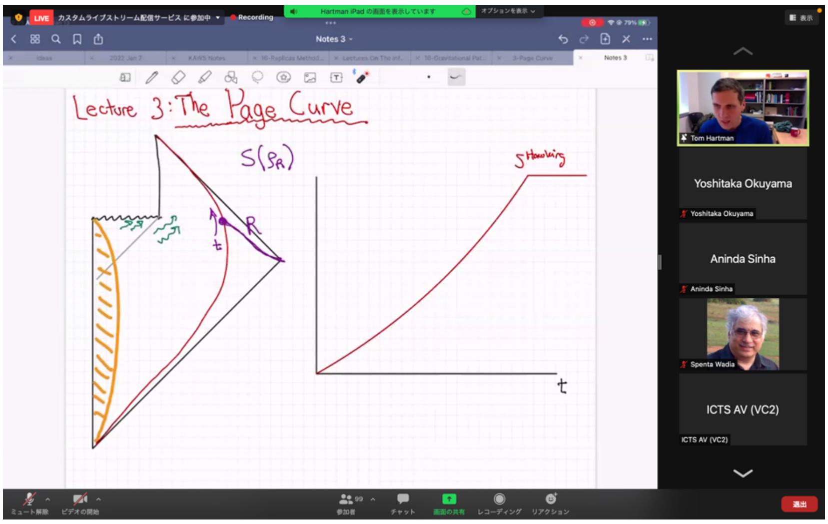
The main topic of his lectures was non-relativistic conformal field theory. The audience, including me, was almost high-energy theorists and was unfamiliar with condensed matter physics. The lecturer was also a high-energy theorist, so he knew exactly where we would get stuck. And he kindly introduced the notion of non-relativistic quantum field theories from scratch. It was stimulus and great fun to learn condensed matter theorists' ways of thinking, where our common sense does not go well.

He began with a simple interacting bosonic model and fundamental ingredients such as dimensional counting, Feynman diagram, spontaneous symmetry breaking, stressing the deviations from relativistic theories. A perturbative (but exact) calculation reveals a non-trivial renormalization group (RG) fixed point when the spatial dimensions are between two and four. After that, he explained how non-relativistic conformal field theory describes the theories on RG fixed point.

Going through specific examples, he discussed the general properties of non-relativistic conformal field theory from a group theoretical viewpoint. The enhanced isometries in non-relativistic systems at criticality are captured by Schrodinger algebra. However, it turned out that we can neither fix the form of three-point functions nor have a concept of state/operator correspondence as in the relativistic case. There is, in fact, an analogous statement of the state/operator correspondence, but it is much different from a relativistic one.

I have been mainly studying fundamental aspects of relativistic conformal field theories. In search of the future direction of my research, like applications to condensed matter physics and S-matrix/conformal bootstrap, all lectures will be a great help.

In the end, I would like to express my gratitude towards all the people concerned for giving me this opportunity.



The third lecture of "Entanglement in QFT and Quantum Gravity."