Symmetry is Everywhere

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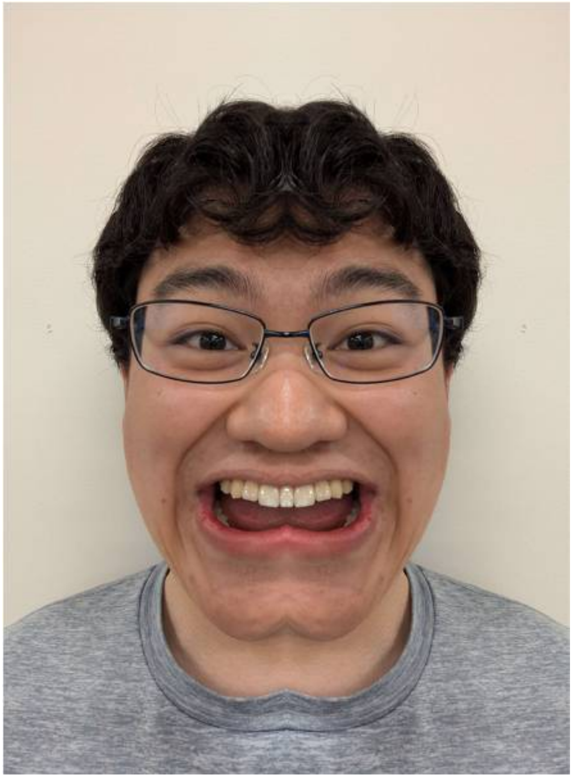

Symmetry is an essential concept in various fields from science to the arts. Many famous buildings, like Tokyo Tower, have bilateral symmetry. In other words, when you see them from the front, the right and left sides are reflections of each other. Another example is the figure above. I modified my face to have bilateral symmetry. Such geometric symmetry is the most intuitive example of the concept of symmetry.

In general, an object has symmetry if it is unchanged under some operations. Physicists consider more general and abstract operations than geometric reflection and rotation: changing the observer. When you get on a Shinkansen or a plane, you cannot know whether you are moving or not without looking outside. This is because known physics laws have Lorentz symmetry, that is, the laws cannot be distinguished by a static observer and another observer who moves at a constant speed. This concept is fundamental in physics. The Standard Model of particle physics, which describes all known phenomena except gravity, is constructed to respect Lorentz symmetry.

Recently, the concept of symmetry in physics has been generalized. All operations considered above have inverse operations. You can undo reflection, rotation, and a change of the observer. However, some operations that cannot be undone are worth considering as symmetry. This new type of symmetry is called non-invertible symmetry. There are many generalizations of symmetry, such as non-invertible symmetry, and they are collectively called generalized symmetry. This is the field I am studying now. I am studying the structure of generalized symmetry and its applications.

You might ask, "So, I understand symmetry is widely used in physics. But why is it useful?" My answer is that symmetry imposes a restriction on the theory, and you can extract much useful information without knowing the details of the system. That is why I became interested in symmetry and decided to work on it.

Let's consider a game called Sugoroku to understand the statement more concretely. You and I start at square 1. We take turns rolling a die and move forward by the number on the die. There are special squares where we must follow the instructions. The first to get to the goal wins, and we restart the game if we reach the goal in the same turn. Then, what is the probability you win?

You can answer this question, although you do not know the details of the system, such as the number of squares or the specific instructions on the special squares. In fact, you do not need to know Sugoroku rules at all. What is important is that you and

I are equivalent in this game, that is, the game has symmetry to swap our positions. Thus, the probability that one of us wins is the same as the other, and turns out to be 1/2. Maybe this question is too easy and boring for you, but it contains the essence of symmetry. You can learn a lot of information about the system only by using its symmetry.

This simple idea appears in physics, where almost all the interesting systems cannot be exactly solved since their rules are too complicated. A few systems that can be solved are called integrable systems. For example, a planet moving around the Sun draws an elliptical orbit if there are no other planets. In non-integrable systems, you must use an approximation to calculate valuable quantities. If you want to calculate the motion of a celestial body precisely, you must include the effect of heavy planets such as Jupiter. Usually, it is assumed that the orbit of the celestial body is not so different from the elliptical orbit in the calculation. However, such an approximation can often lead to wrong conclusions or be applicable only in limited situations. If an asteroid gets too close to a heavy planet, it may be flicked away and go outside the solar system. Therefore, it is crucial to develop methods to get some information without entirely solving the system or introducing an approximation.

Symmetry lies at the heart of such methods. Actually, one symmetry is not enough. If you perform one symmetry operation and another operation, the whole operation is a symmetry operation too. Then you can make a product of symmetry. You can extract information from the relationship between symmetry. The mathematical structure of symmetry is very beautiful and deep, which fascinates me. I believe my study on symmetry will deepen the understanding of many systems in physics.

I found one lesson in learning symmetry. At first glance, things look too complicated to understand. Nevertheless, there is often a hidden principle behind them. Always try to obtain general rules. This is the first advice for you.

However, do not get so absorbed in pursuing an abstract argument that you are unfamiliar with concrete examples. Proper examples will help your understanding. You do not understand the theory if you do not know any examples.

I gave you two pieces of advice on your learning attitude so far. I do not suggest specific fields you must or must not learn, because what I have learned has benefited my life. Moreover, I do not think you will be fascinated if you are compelled to study physics. Actually, I do not care whether what you are interested in is physics or not. I am happy if you find something worth dedicating your life to. It will always guide you in the right way.

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