

Foundations of Quantum Theory: Seeking what *kind* of theory nature obey

Shun Umekawa

I am now majoring in *theoretical physics*, which is the field trying to find fundamental principles of nature. I know that at that time you were more interested in history or mathematics than in science. You may have learned many *facts* about nature, or some phenomenological laws of it, such as the revolution of the moon, the chemical reaction between HCl and NaOH, basic rules of electric circuits, or the classification of plants. However, the act of science is **not** limited to collecting the facts and discovering new phenomena; there is a field of science called **Physics**, which is the attempt to find a small number of basic *principles* of nature which can potentially explain those facts from a universal perspective. If you learn physics, then you can *understand* the nature, not only to *know* the nature. That is why I am fascinated by physics. I guarantee that physics is very different from other fields of science, and you must find it interesting.

In particular, I am studying **foundations of quantum theory**. Quantum theory is a fundamental theory of physics that, we believe, governs everything in nature. The effects of quantumness are conspicuous in microscopic world, but macroscopic objects also obey the laws of quantum theory.

One of the most surprising results of quantum theory is the denial of ontology; the classical view of science that the value of a physical quantity is pre-determined and what we do in experiments is to know it. Though it seems true by our intuition, the discovery of quantum theory revealed that it was wrong. The value of the physical quantity cannot exist independently on measurement context, and we can only predict

the probability distribution of the measurement outcome. Moreover, there exist pairs of physical quantities, e.g. position of a particle and its speed, that cannot be simultaneously determined, i.e. their precise joint measurement does not exist. The important thing is that they are prohibited from the first principles. Indeed, we can rigorously prove that the predictions of quantum theory cannot be reproduced by any ontological models through, for example, the violation of the Bell inequality [1]. This is such a surprising result that, I think, changed the human's fundamental perception of the nature.

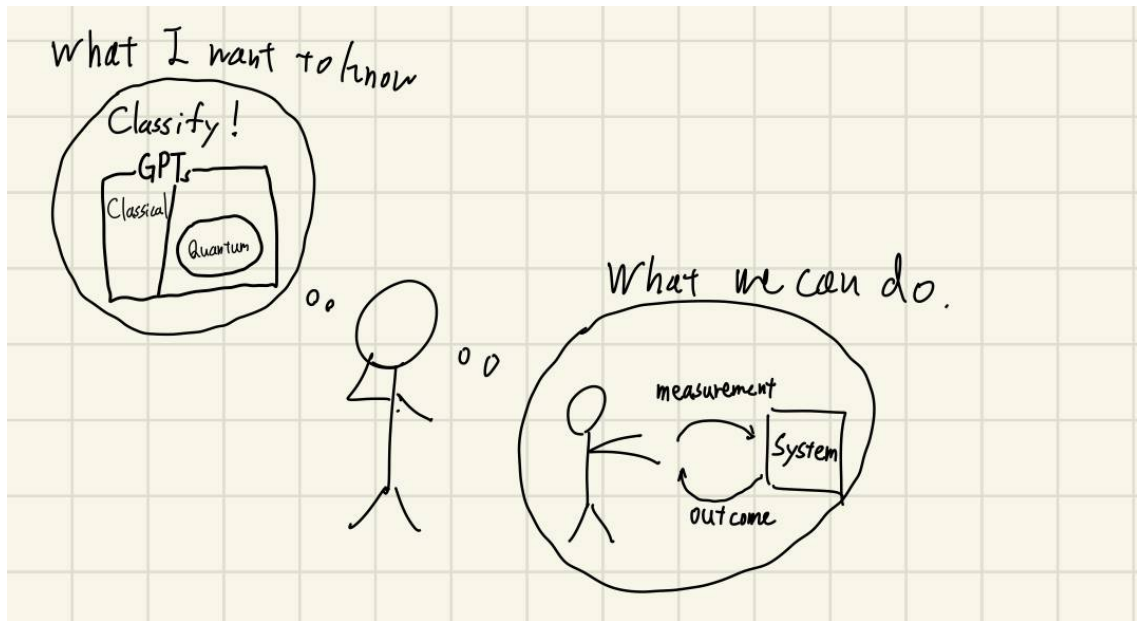
Since quantum theory is the foundation of most theories of physics, I want to understand what the quantum theory is, by revealing its mathematical structure and its universal properties that are not limited to a specific system or situation.

What I am currently most interested in is the topic named **Generalized Probabilistic Theories (GPTs)** [2]. GPTs is a theoretical framework that contains quantum theory and classical (probability) theory as a special case. A remarkable feature of this framework is that it grounds its basis on purely operational axioms regarding states and measurements. For example, we require that if there are two states A and B, then there also exist a state $pA+(1-p)B$, such that the system is in state A with probability p and in state B with probability (1-p), which implies that the state space is closed under convex combination. Since the axioms of GPTs consist only of physically reasonable statement, any (potentially be) physical theory must be a special case of GPTs. (Although most of GPTs are merely hypothetical thing.) Moreover, mathematical concepts definable for general GPTs almost certainly have operational interpretation itself and therefore we can carry an argument only using physically well-motivated concepts.

Within the framework of GPTs, we can detect the origin of features of the quantum theory; identifying which ones are actually common among all non-classical theories, and which are truly characteristic to quantum theory. It is also certainly important to show what we can say within quantum theory, or to clarify the difference between quantum theory and intuitive classical theory. However, in order to truly understand what the quantum theory is, I believe, we need to classify it in a wider framework. That is why I am attracted to GPTs.

As advice from future self, I recommend you to study widely. One thing I learned through these 10 years of experience is that I cannot correctly judge where a field is worth studying for my interest just by the name of the field and rough explanation. I have experienced many times that something sound not so interesting and not relevant for me is actually very interesting and useful for me. Unfortunately, even though the content contains something interesting for me, people do not always explain it in a way that attract me. Some (physicists) say that it is O.K. to study (a field of mathematics) when you need it. I agree that its literal statement is true. However, I am very doubtful whether I can notice that I need it without a little experience of studying it.

Indeed, I realized that this is how the science develops. There have been countless times throughout the history of science that a mathematical concept or physical discovery that looks useless at first glance actually has rich structure and is found to have many applications. In order to move forward, you need to study widely and keep your source of inspiration broad.



[1] “For experiments in entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science”, Scientific Background on the Nobel Prize in Physics 2022 (2022), (<https://www.nobelprize.org/prizes/physics/2022/advanced-information/>).

[2] M. Plávala, “General probabilistic theories: An introduction”, Phys. Rep., 1033, 1 (2023), (<https://doi.org/10.1016/j.physrep.2023.09.001>).

Acknowledgement:

I thank Prof. Kate Harris and Kensuke Honda for helpful comment.

AI tools: I used Chat GPT to correct the grammar and to improve my expression in English.