Quantum Hall Effect and Mathematical Physics

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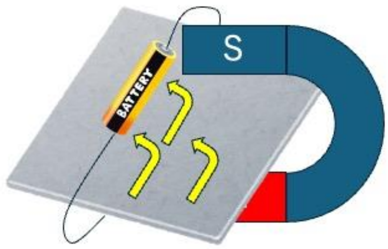


Condensed matter physics is a field of physics that deals with the properties of matter such as electric resistance or specific heat. Within this field, one-dimensional systems such as linear materials or rings have a lot of special properties. One of them is the solvability of models. In condensed matter physics, we sometimes consider some mathematical model that reflects the features of real matter in a simple way. For example, to research the behavior of magnets, we construct models in which many tiny magnets called "spins" are aligned on a lattice. The lattice corresponds to atoms and spins represent the source of the magnet of electrons on each site. Each spin affects the others so that the direction of them becomes the same. By analyzing how the spins are aligned, we can understand the mechanism of magnetism and predict the behavior of magnets in various environments such as high temperatures. Not limited to magnetism, a lot of models are constructed corresponding to the properties of interest. In most cases, we have to deal with a lot of number of particles interacting with each other, so it is difficult to calculate the behavior of the models exactly. However, for some models, we can obtain the main properties of the model exactly. These models are called solvable models. In one-dimensional systems, it is known that some models are solvable even though they are many-body problems. In this field, I would like to research the properties of such models. Especially, how these models react to the external field such as the electric field is not completely figured out.

What made me to be a researcher of condensed matter physics was learning about the quantum Hall effect. When I was an undergraduate student, in a university program, I was assigned to a seminar in condensed matter physics, which I was not interested in, and I learned about it there. The setup for the quantum Hall effect is a thin film of materials that conduct electricity applied to a magnetic field perpendicular to it. A magnetic field is generated by a magnet and its direction is from the S pole to the N pole. Because of the properties of a magnetic field, if we apply voltage to the system, current flows not only in the same direction as the voltage is applied but also in a direction perpendicular to it and causes a voltage in that direction as well. This is the Hall effect and the ratio of the current flowing in the original direction to the voltage perpendicular to it is called the Hall conductance. Usually, as the magnetic field becomes stronger, the Hall conductance is proportional to the magnetic field. However, if we cool the system to low temperatures, the behavior of the Hall conductance is changed dramatically. The value of the Hall conductance changes like a stair to the magnetic field, and at a flat part, takes some integer multiple of a certain value [1]. Roughly speaking, the value of the Hall conductance is limited to the integer multiple of a certain value. This is called the quantum Hall effect and is described by quantum mechanics. In our daily lives, physical properties such as the weight or height of our bodies and the intensity of light can change continuously, so it is counterintuitive that a property of matter is limited to discrete values. I was surprised by this phenomenon and studied the theoretical background of it. Then, I was impressed that the result of my calculations with my hands matched the phenomenon, and wanted to find other such phenomena by myself. Also, the quantum Hall effects have a deep relation with topology. Topology is a field of mathematics, that treats the invariant value through continuous transformation. For example, consider a balloon that can transform freely. We can deform it to a sphere or cube, but we cannot deform it to a torus without blowing it. Similarly, a balloon in the shape of a torus cannot transform into the shape of a sphere. In this sense, the number of holes in the shape is invariant through continuous transformation. In the quantum Hall effect, the value of Hall conductance corresponds to the number of holes. I have been interested in mathematics since I was a child, so I was strongly intrigued by the mathematical structure behind the quantum Hall effect and wanted to know more about the relationship between other physical phenomena and mathematics. In short, I was impressed by the quantum Hall effect and its mathematical structure and became a researcher with an ambition to discover such phenomena by considering both mathematics and physics.

After the discovery of the quantum Hall effect, and understanding it, topology was actively brought into physics, and a new field of physics was explored. Since physics is described in terms of mathematics, we can see a lot of interesting mathematical structures hidden in physical phenomena. My research may find a new interesting structure in physics and open a new type of mathematical physics and other fields of science such as machine learning. Also, historically, condensed matter physics has been closely related to our lives. For example, the studies on semiconductors are applied to transistors, which are indispensable for smartphones and computers. Also, superconductors are used in linear super-express. Similarly, my research may be able to make our society convenient.

This essay was assisted by DeepL to translate some phrases and by Grammarly to check grammatical errors.

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References

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