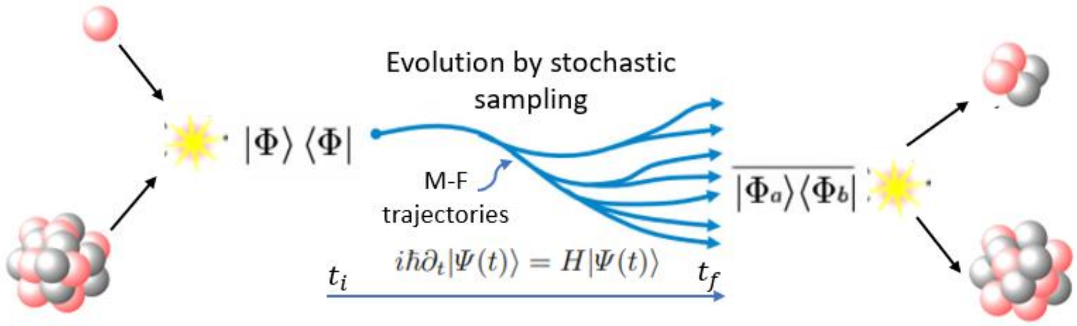
Stochastic method and interpretation of deterministic evolution

ZHANG Zhaozhan



What's the building block of our material world? Since ancient Greece times, people have been trying to answer this question and find appropriate language or pictures to describe them. Today, we know that a drop of water can be composed of water molecules, which can be composed of hydrogen and oxygen atoms. An atom is composed of a positively charged nucleus and some negatively charged electrons. The atomic nucleus consists of chargeless neutrons and positively charged protons. Of course, this story can continue to go on, and we can keep breaking up the nucleons to find more fundamental particles.

For a nuclear system at the size of a few femtometers, the quantum effect becomes important. The system is described by a wave function $|\Psi\rangle$ and its evolution follows the Schrodinger equation, as illustrated in the above figure. But even if we can somehow explicitly write down the underlying physics, it is difficult to solve it and figure out how they interact and how this interacting system evolves. One difficulty is that the degrees of freedom to treat are enormous, which implies that a finite computer is needed to solve problems that can easily become nearly infinite in dimensionality. Indeed, the nuclear system itself is a quantum many-body interaction system, which can be composed of up to a few hundred nucleons. To describe the real dynamic of this system, it is necessary to consider and couple all the degrees of freedom of the many-body components, which will make the problem quite complex and usually out of computing power. In nuclear physics, a general strategy to solve this problem approximately is to decouple the many-body component with a non-interacting picture. This approximation also called the mean-field (M-F) method, has succeeded to approximate and explain many experimental phenomena. However, methods beyond the mean-field level are required to reproduce some correlation effects, such as nuclear collision, pairing, and fluctuation. Recently, a new interpretation of the nuclear dynamic makes it possible to combine the mean-field and stochastic methods [1]. To explain more clearly this method, we will first turn to the mean-field picture in the following part and finally return to its stochastic interpretation.

Imagine one simple situation where a system is composed of non-interacting components. To describe the dynamics of the whole system, it is sufficient to follow the dynamics of each single component individually since each component evolves independently. In quantum language, this means that the wave function $|\Psi\rangle$ of the whole system can be written into a product wave function form. When considering the interaction, such as the two-body interaction, each component can couple with each other, and the wave function's form may become complex. Instead of a single product form, it can be written into their superposition. But in the lowest approximation, it is possible to restrict the solution with a single product form and find the most appropriate one, which may be the basic idea of the mean-field theory in nuclear physics. In this case, the interacting nuclear many-body system can be approximated by a collection of independent movements in an average field generated by all the other components. This method has been greatly used in nuclear physics to grasp some main characteristics of nuclear dynamics. But there is still limitation due to lack of coupling between single components degrees of freedom and it has partly considered the interaction by an effective external field. To fully consider the correlation between different components, methods beyond the mean-field level are required.

In the year 2002, researchers reformulated the dynamics of the nuclear many-body problem interacting with a two-body interaction [1]. They found that it is possible to reproduce the exact dynamics by a series of mean-field trajectories following some distribution, as illustrated in the above figure. In other words, the trajectory of the exact dynamics can be simulated by stochastic evolution of mean-field trajectories. The deterministic evolution can be simulated by a series of stochastic evolutions. This stochastic evolution has some analog with phenomena in other fields, such as Brownian motion described by the Langevin equation. In this analog, the single components in the mean-field trajectory are somehow like the random Brownian motion suspended in a medium. It is thus possible to bring new perspectives from other fields. Since then, various attempts based on this new framework have been proposed to go beyond the mean-field level and further understand the nuclear dynamics and the correlation effect. This technique has been also applied in other fields such as open quantum context to decouple the environment and system correlation [2].

Finally, apart from the new interpretation of nuclear dynamics, why this reformulation will be of interest? Indeed, this can be explained by a few aspects. For practical calculation, the standard mean-field codes exist already for years. Therefore, it is possible to perform several independent stochastic evolutions with much less degrees of freedom in several computers, which should be more efficacy than the handling of a matrix with large dimensions. Secondly, the analog with phenomena of other fields such as the study of Brownian motion may induce new perspectives and new techniques to the understanding of nuclear physics.

The essay was written without the use of assistive technologies. The author wants to thank Takahiro Sato for his careful reading and useful propositions for this article.

Reference:

[1] O.Juillet, P. Chomaz, Phys. Rev. Lett. 88, 142503 (2002)

[2] J. Shao, J. Chem. Phys. 120 5053 (2004)