Entropy: What We Can and Cannot Do

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1. Introduction

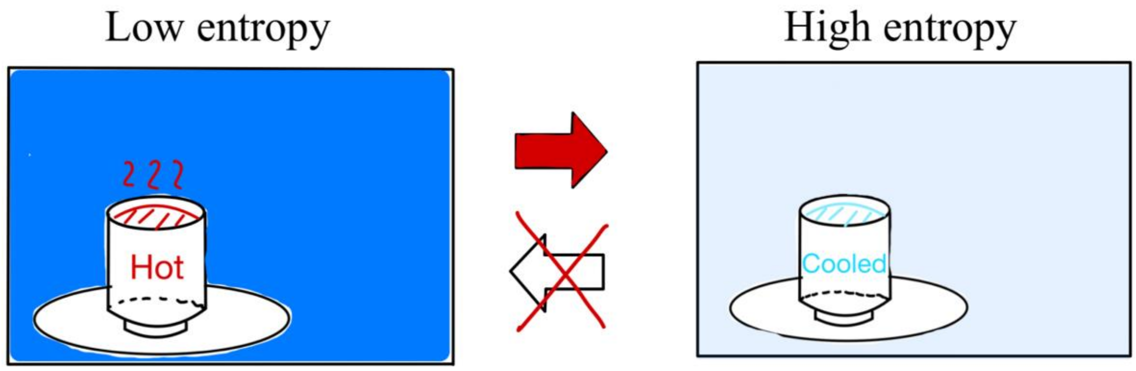
Have you ever heard the term entropy? Many people use the term as an intelligent paraphrase of clutter: “My room is cluttered because of increased entropy! It's the natural order of things!”. This usage is certainly not wrong, but entropy is essentially a term from thermodynamics, a branch of physics which treats heat and temperature. Historically, this concept was defined after a difficult argument. However, in 1999 H. Lieb and J. Yngvason(1) succeeded in defining this concept in a very simple way. This is not just a simplification but has important physical implications that still have an impact across disciplines.

2. The Concept of Entropy and What Was Discovered by Lieb and Yngvason

Thermodynamics is a study that deals with concepts such as heat and temperature. One of the starting points of the discipline was the empirically known fact that heat cannot be transferred from cold to hot matter without leaving behind some other change. This is called Clausius statement. This statement and equivalent claims are collectively referred to as the second law of thermodynamics. Clausius statement says that hot water may cool to room temperature, but cooled water does not spontaneously return to hot water. In short, the transfer of heat is irreversible. Can we define a quantity that characterizes this irreversibility? This is precisely the entropy, and the second law of thermodynamics can be stated using entropy as follows.

When no heat goes in or out, the total entropy does not decrease.

In the previous example, a state of small entropy (hot water and the room) changes to a state of large entropy (Water and the room at the same temperature).



Then, how is entropy defined? If I were to present a history of how it is constructed, since the logic is so complex this section would be the size of a textbook. However, in 1999 H. Lieb and J. Yngvason showed that entropy can be constructed from very intuitive and straightforward assumptions. The core concept of their work was *adiabatic accessibility*. This is a relationship between states that describes whether a system can change from one state to another only by raising or lowering a weight attached to the system. The word “adiabatic” means “no loss or gain of heat”, but as mentioned above, adiabatic accessibility is defined without the concept of heat! Making some natural assumptions about this relationship, they redefined the measure of irreversibility, or *entropy*.

3. How Our Understanding of Entropy Has Changed

To explain the excellence of their work, we first see what was so complicated about the historical definition of entropy. It is expressed as heat divided by absolute temperature (or precisely, its integral). While this seems straightforward, there are two difficulties here. One is the question of how to define absolute temperature. One method is to admit a priori an empirical temperature and construct the absolute temperature as a function of it. Another question is whether this quantity is defined independently of the process of state change. The amount of heat absorbed or released depends on how the state changes, so we need to be sure that the above definition is independent of the process of change. Traditional textbooks on thermodynamics devote a lot of paper to discussing the above two points.

How did H. Lieb and J. Yngvason's definition resolve these difficulties? They did not deal directly with concepts such as heat and temperature but started from the adiabatic accessibility mentioned in the previous section. In essence, they focused on irreversibility itself. They made several simple assumptions such as invariance to addition of auxiliary systems and scaling of systems. The important point is that these assumptions are sufficient for the definition of entropy, and concepts such as heat and temperature, for example, are not necessary. (Of course, these can be defined later, if necessary.) The beauty of this work is that entropy, which had been a theoretical and secondary quantity associated with heat and temperature, was seen to be a universal, fundamental, and intuitive quantity derived from irreversibility itself.

4. Spread to Other Fields - Focusing on Resource Theory

The redefinition of entropy by Lieb and Yngvason has implications beyond thermodynamics. Abstractly speaking, they have characterized how states can be transformed when the operations to be performed are fixed. This abstract framework leads to resource theory(2), which consists of objects and free operations that represent transitions between them. One of the goals of this theory is the construction of useful “monotone functions”. This means a function of states that does not decrease with free operations. This is the very generalization of entropy. Once this function can be computed, our operational limit, or general “second law” is known; we cannot reduce its amount freely! Although these concepts seem seemingly too abstract to be useful, it can explain limitation in familiar technologies. In thermodynamics, the second law govern the limit of efficiency of heat-to-work conversion, which is closely related to power of steam turbine in generator. In information theory, there is a limitation of data compression, which explains how much data (or “giga”) is required at least to talk on a phone, see a video in YouTube, and so on. Of course, these results are discovered independently, but resource theory can give unified understanding.

In summary, Lieb and Yngvason show that entropy can be derived by focusing on irreversibility itself, and their ideas extend across disciplines. The second law of thermodynamics represents the limit of *what we can and cannot do*, which is precisely the topic of interest in the field of information processing.

As technology rapidly develops and what we can do changes daily, it becomes increasingly important to consider and quantify what we can and cannot do. By discovering the limitations of current technology through basic science, technology will develop to approach or overcome them. Even for familiar problem such as “making Internet communication speed faster”, basic science can give important limitations. Let's learn basic science and find out what we cannot and will be able to do!

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