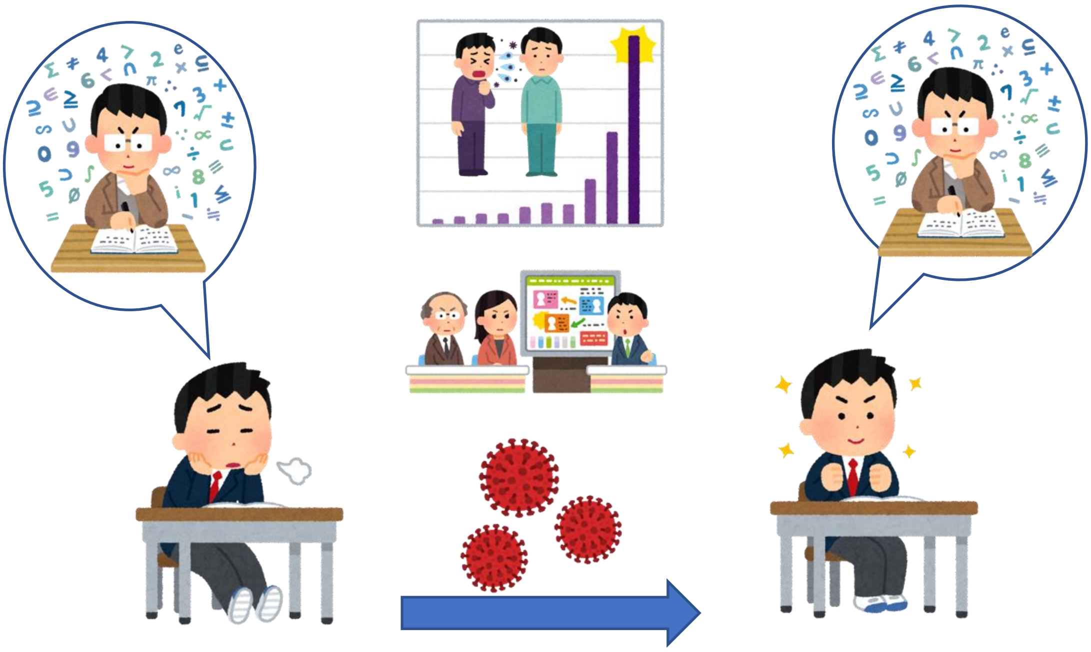
The model of COVID-19: the equations which affect and attract people in all fields

Uchida Yoshihito



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coronavirus: https://www.irasutoya.com/2020/01/blog-post_265.html

mathematician: https://www.irasutoya.com/2016/04/blog-post_157.html

droplet infection (epidemic): https://www.irasutoya.com/2020/07/blog-post_38.html

overshoot, explosive infection: https://www.irasutoya.com/2020/03/blog-post33.html

news, long variety program: https://www.irasutoya.com/2017/02/blog-post_444.html

unmotivated student: https://www.irasutoya.com/2020/01/blog-post_80.html

motivated student: https://www.irasutoya.com/2020/01/blog-post_68.html

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Some of you may think that mathematics and any other fields of science are not useful for our lives directly, and others consider the researcher in these fields as a freak because they spend a lot of time (some of them spend all their lifetime) on studying what they want to understand, following their interest. However, basic science sometimes dominates unexpected fields of our lives and national system, such as economics and politics. In this essay, I'll introduce you a mathematical model which has great influence, in the COVID-19 pandemic, on politics and consequently on our lives.

As you know, the number of people with COVID-19 were reported every day and the estimation of it had great influence not only on medical scientists but also on national government and any other non-researchers. For example, the government decided whether to declare a state of emergency or not and cancel it based on the estimation. When you estimate the number of those who are infected with COVID-19, you use a mathematical model which is also related to physics: SIR model [1]. Even though this model was proposed about 100 years ago, it had been discussed every day in the COVID-19 pandemic. It is often said that the number of those infected are expected to increase exponentially, which you may have heard on TV or watched some figures which show it. This result can be reached by analyzing the model, for instance [2].

SIR model is a simplest model which predicts temporal development of the number of Susceptible (those who have not been infected but may become infected), Infectious (those who are infected), and Recovered (those who were infected and are immune to COVID-19, i.e., they will never be infected). This model includes the following hypotheses:

(a) Susceptible and Infectious meet, and then Susceptible becomes infected.

(b) Infectious are recovered at some point, that is, Infectious never dies.

This model also assumes that the total population does not change, that is, it does not consider the possibility of someone's death due to COVID-19 or other diseases, birth of a child, etc. Also, those who have already been infected are never become infected in this model (If you want to know the detail of this model, please refer to the footnote on this page[1]). Some of you may think this model is too far removed from reality, but this simple model gives us fruitful results and essential properties of phenomena (The exponential increasing of Infectious is one of the examples.), and thus it is interesting for original basic science researcher, who pursue the essence of something in natural world.

SIR model is the simplest case of Kermack-McKendrick theory (1927) [1]. It is

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[1] The equations of the model are (1)-(3) and what they mean is as follows:

$$\frac{dS(t)}{dt} = -\beta S(t)I(t) \ \cdots (1)$$

$$\frac{dI(t)}{dt} = (\beta S(t) - \gamma)I(t) \cdots (2)$$

$$\frac{dR(t)}{dt} = \gamma I(t) \ \cdots (3)$$

The left side of those equations is the temporal transition of the number of S (Susceptible), I (Infectious), and R (Recovered) at a certain time. The right side of (1) and the right side first term of (2) mean that Susceptible and Infectious meet, and then Susceptible becomes infected. So, the number of Susceptible decreases and that of Infectious increases as time goes by. The second term of (2) and the right side of (3) mean that Infectious are recovered, and the number of Infectious decreases and that of Recovered increases consequently. Here, β is the infectability of COVID-19 (e.g., a larger value of β means that the infectious capacity is stronger) and γ is the quantity which is correspond to the recuperative powers of Infectious (e.g., a smaller value of γ means that infectious recover from COVID-19 more quickly).

very natural to think that we can evaluate the magnitude of each epidemic (e.g., the number of Infectious) by understanding the causal element or the mechanism of each epidemic, which is the motivation of their research. To tackle this challenging problem, they thought that mathematics is necessary to make the essence of epidemics clear and proposed the much more generalized model than the work by Ross and Hudson (1915-1917). They considered seriously the apparently obvious fact that the infectiousness to Susceptible changes as time goes by because the stage of sickness also changes as time passes. The mathematical structure of this generalized model is so complicated that many mathematicians has tried to analyze it and understand everything of the model. Finally, in 1970s, Diekmann, Metz, Theime and others succeeded in understanding the whole structure of the model. But, once we consider a bit more complicated case, we cannot say that we have already understood all, and there is room for more detailed analysis.

SIR model has impact not only on mathematicians or medical scientists but also physicists (and other basic scientists) because this model is closely related to the model of radioactive decay, that is, the form of SIR model is almost the same as the differential equations which describe some of the radioactive reaction. Speaking of radioactive reaction in our lives, the Great East Japan Earthquake and the subsequent nuclear power plant accident in 2011 are still fresh in our memory. Even now, we are not allowed to enter some areas near the power station. Why is that? We can understand it by analyzing the model of radioactive decay. The model is similar in form to SIR model, and thus we can obtain the conclusion that the intensity of radiation from radioactive materials decreases exponentially, that is, if a half-life of certain radioactive material is 30 years, it takes more 30 years to reduce the intensity to a quarter of the original level. This means that the intensity of radiation continues to remain strong over a period of our life span, and thus those who had lived there cannot return home. Such a horrifying accident because of earthquake disaster and a serious epidemic are terrible for our lives, but it is very interesting that these are related to each other from the scientific point of view. Moreover, a result from mathematics has affected not only mathematicians but also researchers in other fields of basic science and our lives.

The interesting point of basic science is that a result from this field becomes useful and important when you least expect it. The estimation deduced from SIR model or something like had influence on government policy and changed our lifestyles. Also, this model has played the role of giving the chance to discuss mathematical problem with people who have different kinds of background, which has led many people to noticing the possibility of basic science. I believe this model will change the bad situation that young students come to hate mathematics and natural science and they are discouraged from studying basic science.

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[1] Kermack, W. O., & McKendrick, A. G. (1927). A contribution to the mathematical theory of epidemics. *Proceedings of the royal society of london. Series A, Containing papers of a mathematical and physical character*, *115*(772), 700-721.

[2] 桂田祐史 (2023)「SIR モデルについてのメモ」

http://nalab.mind.meiji.ac.jp/~mk/labo/text/sir.pdf（accessed 2023-05-15）

[3] 稲葉寿. (2008). 微分方程式と感染症数理疫学. *数理科学*, *538*(1), 7.