We Live in Turbulent Flow

35-196049 Yohei Zushi, August 2020

Turbulent flow or turbulence is irregular and unsteady flow. It contrasts with laminar flow, or steady flow. As a simple example, let us consider a water tap. If you open the faucet a little, the water falls straight down, but if you increase the flow rate, the water flow become violent and irregular. The former is laminar flow and the latter is turbulent flow. There are many other types of turbulence around us. In fact, most fluids in our daily lives, such as air and water, are turbulent. For instance, atmospheric flow and building wind are turbulent, and the air in a room stirred by an air conditioner or a fan is also turbulent. In addition, river and ocean currents are turbulent, and dynamic phenomena such as eddies are also seen. Although turbulent flow is a familiar phenomenon in everyday life, the science of turbulence remains unsolved. Evidently, turbulence is too complex to treat theoretically and numerically with current research methods.

Humans have been living with water since ancient times, and research on fluid dynamics has a long history. Archimedes, an ancient Greek scholar, made discoveries about buoyancy, according to a famous anecdote, with the exclamation “Eureka!” He was also said to have developed a screw pump with a helical structure. In the Renaissance,

Fig.1 self-portrait and turbulent flow around piers by Leonardo da Vinci
(picture from Wikipedia)

Leonardo da Vinci, a polymath, studied the flow of water and made sketches of turbulence around piers in a river (**Fig.1**). He observed eddies, or vortices, in the flow. As for the theoretical treatment of fluids, in the 18th century when Isaac Newton had established classical mechanics, Leonhard Euler and Daniel Bernoulli introduced the field of fluid mechanics by applying Newtonian mechanics to fluids. However, the theory at that time ignored viscosity, and was far from an actual fluid. To be more specific, if you moved in this fluid, you would not feel its resistance, which is different from our real experience. In the 19th century, Henri Navier and George Gabriel Stokes introduced the equation of motion of viscous fluids, which made it possible to handle more realistic fluids. One of the most famous turbulence experiments was performed by Osborne Reynolds in 1883, who investigated the laminar-to-turbulent transition of fluid in a pipe. He found that the flow could be characterized by dimensionless parameter, which is now called the Reynolds number, $Re = UL/\nu$, where U is the velocity of flow, L is the characteristic length, and ν is the kinematic viscosity. By the observation of ink flowing in a pipe, Reynolds discovered that when the velocity, i.e. Reynolds number was increased and exceeded a certain value, the flow of ink changed from straight laminar flow to disordered turbulent flow.

Turbulence is a dynamic phenomenon far from an equilibrium, which is relatively easy and well understood, and there is no universal theory to date to explain it. To illustrate this deficit, the Navier-Stokes equation, which is the equation of motion for a fluid with constant density is represented by

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\rho}\nabla p + \nu\Delta\boldsymbol{v},$$

where $\boldsymbol{v}$: velosity, t: time, ρ: density, ν: viscosity.

The left-hand side represents the acceleration of the fluid along the flow rather than at a fixed point. The first term of the right-hand side is the pressure-derived force and the second term is the viscosity-derived force. If we can solve this, we should be able to find out exactly how the motion of the fluid is changing with time. However, this equation is nonlinear and difficult to solve. Moreover, it is not known whether there is a general solution or not. We can impose special constraints to derive a solution, but it will be different from turbulence. Whether there is a smooth, non-singular solution of this equation given the initial conditions ("Navier-Stokes existence and smoothness") is a Millennium Prize problem posed by the Clay Mathematics Institute in the USA. If you can solve this problem, you can win one million dollars.

Due to the difficulty of handling this problem analytically, numerical approaches have been taken, they are also challenging. In general, for a numerical simulation, space is discretized into meshes, or many small regions. However, one of the characteristics of turbulence is the existence of vortices, which are also shown in da Vinci's sketch and exist on a wide scale of size from large to small. When we try to calculate even small vortices accurately, it requires cutting meshes so finely and calculating so many meshes that the simulation can be carried out only using a supercomputer. Even so, flow cannot be accurately predicted down to the smallest detail. Therefore, the current theoretical approach to turbulence involves further modeling the Navier-Stokes equation and extracting its features.

A better understanding of turbulence, and its prediction and control, will be of great significance to many fields in addition to fluid dynamics, such as meteorology, geoscience, and astronomy. Turbulence can also be treated as a phenomenon called chaos, which is governed by deterministic laws, but whose long-term predictions are currently impossible.

A typical example is a double pendulum. Understanding turbulence may give us hints when dealing with such chaotic systems. Furthermore, given the many examples of turbulence around us, there are many potential benefits to society with a greater scientific understanding. For example, accurate predictions of atmospheric turbulence would allow near-perfect weather forecasts. In addition, turbulent flow is characterized by a large energy dissipation with strong effects on diffusing and transporting heat and substances. If turbulent flow is suppressed, energy savings in a pipeline, for example, are expected. In contrast, if turbulent flow is promoted, fuel gas and air can be better mixed in an engine for greater efficiency. Turbulent flow is also used to reduce noise or air resistance in high-speed vehicles such as cars, bullet trains, and airplanes, and it is expected that such functions will be enhanced. In this way, understanding and controlling turbulence will be very useful for technological progress.