The Arrow of Time: Bridging the Macroscopic and Microscopic Realms

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August 2020

Thermodynamics and quantum mechanics are unshakable pillars of modern physics, that deal with two respective hierarchical levels of nature, the macroscopic and microscopic. Thermodynamics describes macroscopic phenomena irrespective of the microscopic details, and its second law is sacrosanct in physics. In contrast, quantum mechanics has revealed the microscopic world's astonishing properties that form the basis of modern technologies. Quantum mechanics requires exponentially many parameters in the number of constituents to describe a system, while thermodynamics requires only a few parameters, whose number is independent of the size of a system. The attempt to bridge these two theories and their respective levels is a major unsolved problem in a field called statistical mechanics.

Statistical mechanics was conceived in the late 19th century by Boltzmann, and Gibbs introduced and established a probabilistic description of statistical mechanics in his 1902 book [1]. Of course, these works were intended to derive macroscopic phenomena from classical mechanics laws, but their extension to quantum systems was made shortly after the discovery of quantum mechanics. Although the usefulness of probabilistic descriptions is established by a century of applications, its scientific foundations in microscopic theory remain unsolved since nature does not evolve probabilistically. This conundrum of bridging macroscopic and microscopic theory has eluded even the deepest thinkers for more than a century.

After a sufficiently long time, a macroscopic system irreversibly evolves into an equilibrium, where any macroscopic change can no longer be observed, a process called thermalization. In statistical mechanics, we assume that the state of a system is random and calculate average values over it. On the other hand, if an isolated quantum system (i.e., a quantum system which is not interacting with the environment) is initially in some state, it remains in some (possibly other) state, which we can calculate with quantum mechanics. That is, the system is always in a definite, and not random, state. The first attempt to reconcile this paradoxical issue was made by von Neumann in his seminal paper [2] published just a few years after the discovery of quantum mechanics. However, owing to criticisms based on a misunderstanding of his main result, the paper was rapidly forgotten. It was not until more than half a century later, in 2010, that researchers rediscovered his paper and correctly understood the importance of its result. This rediscovery was brought out by recent progress in experiments, which enabled the isolation of a quantum system with unprecedented accuracy, and the thermalization of such systems could be observed. These results stimulated a resurgence in the foundations of quantum statistical mechanics and a reassessment of von Neumann's paper.

Although von Neumann's work had been forgotten until 2010, his idea to consider properties shared by almost all microscopic states, which is now called the typicality argument, turned out to be quite significant, and recently, more sophisticated theories are based on it. Indeed, equilibrium is now understood to be a property shared by an overwhelming majority of microscopic states. The point is that we do not need any randomness, and even a single state is enough to extract macroscopic properties of a quantum system.

Notably, this picture gives an explanation to the problem of the 'arrow of time,' which is a major difference between the microscopic and macroscopic world. The problem of the 'arrow of time' is as follows: in microscopic theory, there is no preferred direction of time, and if some motion occurs, its reverse is always possible. However, in the macroscopic world, there are many processes that cannot be reversed. For example, if we bring into contact a hot material and a cold material and wait for a long time, the temperatures of the materials finally become the same, but its reverse cannot occur unless we force it. The typicality argument reconciles this difference by revealing that the states in which the temperatures of the materials are different are extremely rare compared to those in which the materials have the same temperature, and thus we expect that the former naturally changes into the latter, but the inverse will not take place (Figure 1).

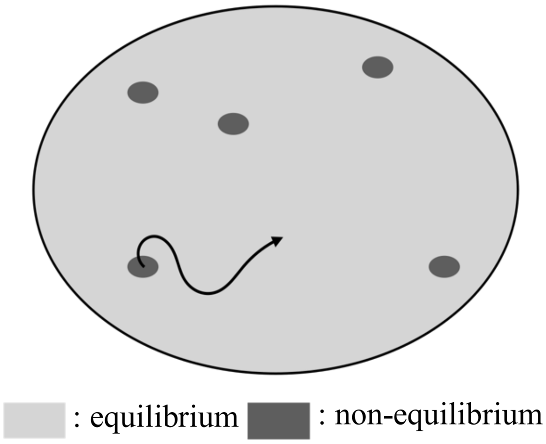

Figure 1. 'Typical' equilibrium states vs 'rare' non-equilibrium states.

An *overwhelming majority* of microscopic states looks macroscopically the same. This majority corresponds to an equilibrium state.

The above argument seems plausible, but it does not ensure that a system prepared in a non-equilibrium state actually relaxes to an equilibrium state. There can be a case that a system remains in non-equilibrium states even though this is extremely rare. The eigenstate thermalization hypothesis (ETH) [3,4] was devised to ensure the relaxation to an equilibrium state and is considered to be a primary mechanism. Here, eigenstates are special states which characterize a quantum system. The ETH asserts that every eigenstate of a quantum system belongs to an overwhelming majority corresponding to an equilibrium. Many macroscopic relations can be deduced from it. However, a rigorous proof of the ETH is elusive.

There are two common approaches to the justification of the ETH, neither of which is satisfactory. One approach is to numerically test the ETH for physically realistic model systems [5]. However, one cannot tell with this approach how generally the ETH applies to physical systems since individual models are inevitably specific. The other approach is to apply the typicality argument to a set of physical systems [6] and extract properties common to many of them. The main difficulty of this approach resides in how to appropriately specify which kinds of physical systems should or should not be taken into account in considering macroscopic theory.

Because there always exist exceptional systems for thermalization, the typicality argument, which attempts to ignore such macroscopically irrelevant exceptions properly, is a promising way towards deriving a macroscopic theory from a microscopic one. To overcome the difficulties in this approach, we will need to consider what operations we can do and how a restriction in our ability to controlling physical systems can affect our descriptions of nature. The realization of this theory will be a formidable task and require a new kind of mathematics that has not been utilized in physics yet. However, if we obtain such a theory, it could significantly extend the applicability of current theories, and we will be able to systematically address the properties of a much broader class of systems. Besides, since statistical mechanics essentially deals with reducing a detailed description requiring many parameters to one with only a few parameters, the sought-after framework may help us to understand the mechanism of machine learning, which also contains large degrees of freedom.

References

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