A Report on Computer Simulation of 2012 Haida Gwaii
Tsunami Generation and Propagation
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Abstract

We simulated tsunami generation and propagation caused by 2012 Haida Gwaii Earthquake occurring on the west coast of Haida Gwaii, British Columbia. Okada's program was used to simulate tsunami generation with parameters of single-fault model, and staggered grid system was adopted with spatial and time grid size of 5 arc-minutes and 5 s respectively. We ran the simulation of tsunami propagation up to 24 hours. It propagated across the Pacific Ocean and had a limited influence on Japan. The comparison between simulated data and DART records was made. We found that simulated waveforms have some consistency with observed waveforms but there still existed difference because of the real shape of elliptic Earth, the effects of the elasticity of the sea bottom and the compressibility of seawater. Finally, we tried multi-fault model to improve our simulation results.

1. Introduction

A tsunami is a series of oceanic gravity waves generated by submarine or coastal geologic processes such as earthquakes, landslides, or volcanic eruptions and could cause serious disasters to human beings. In this report we would mainly focus on tsunamis caused by seafloor displacement due to large shallow earthquakes along subduction zones.

Analytic solutions for linear gravity waves generated from several types of sources could be obtained by Green’s functions. [1] If the wavelength of the seafloor displacement is much larger than the water depth, it can be assumed that the water surface displacement is the same as the seafloor displacement. Usually, only the vertical components of seafloor displacements are considered during the analysis of a tsunami’s generation.

The tsunami propagation depends on the water depth and numerical computations on actual bathymetry have been popular made via the staggered (leap-frog) grid system. The boundary condition used is a land-ocean boundary, in other words, the assumption of a total reflection of energy on the coast. The Deep-ocean Assessment and Reporting of Tsunamis (DART) records water levels using bottom pressure gauges and sends signals to a surface buoy via acoustic telemetry in the ocean and then via satellites to a land station in real time. [1] The observed waveforms, or the data measured by DART could be compared with computer-simulated waveforms and their consistency could be checked through comparison.

2. Haida Gwaii Earthquake

The 28 October 2012 $M_w$ 7.8 Haida Gwaii Earthquake occurred off the west coast of Haida Gwaii, British Columbia. Plate motions in the region were primarily taken up by strike-slip faulting and the seafloor uplift generated a tsunami, whose runup exceeded 3 m and the maximum was 13 m according to field surveys of deposits along the west coast of Haida Gwaii. [2]

Fortunately, no major structural damage was

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reported from any of the population centers in the vicinity. No casualties or major injuries were recorded from the quake, likely due to the sparsely populated nature of the region.

A single-fault model was used by Aditya Gusman for the 2012 Haida Gwaii earthquake. Firstly, seismic W phase inversion was adopted to determine earthquake strike 314°, dip 25° and rake 100°. Then the fault depth, slip amount, fault length and fault width were adjusted to obtain good agreement between observed and simulated tsunami waveforms at the DART stations. The final single-fault model had the shallowest part of fault at 1 km, slip amount 2.5 m, fault length 110 km, and fault width 50 km. The location of shallowest northwest corner of the estimated fault model was at 52.1°N and 131.7°W. [2]

3. Method of Simulation
Tsunami simulation consists of two main parts: simulation of tsunami generation and simulation of tsunami propagation.

Since we have obtained the fault parameters of Haida Gwaii Earthquake, we could calculate the surface displacement on the seafloor. Efforts to develop the formulations in a more realistic earth model have been advanced through numerous studies, but the analysis of actual observations is mostly based on the simplest assumption of an isotropic homogeneous half-space and the simplest source configuration. [3]

Yoshimitsu Okada presented a complete suite of closed analytical expressions for the surface displacements, strains and tilts due to inclined shear and tensile faults in a half-space for both point and finite rectangular sources. By using Okada’s model, the displacement on seafloor surface could be calculated from fault parameters. In Figure 1, the red lines represent uplift while the blue lines represent subsidence. The yellow rectangle represents the single-fault with a length of 110 km and a width of 50 km. In addition, the focal mechanism of the earthquake and the aftershocks in the following one month are presented in the figure as well.

Figure 1: Displacement on seafloor surface calculated via Okada’s model. Red lines represent uplift while the blue lines represent subsidence.

It is necessary to mention that the Haida Gwaii earthquake is more similar to dip-slip earthquake rather than strike-slip earthquake as the rake is 100°. After calculating the seafloor displacement, we assume that the slip occurred instantly, so the final seafloor displacement was actually similar as the displacement of the water surface, which could be used as the initial condition for tsunami propagation. [4]

When it comes to the simulation of tsunami propagation, we should describe particle motions in fluid dynamics properly. Lagrangian approach and Eulerian approach are two different ways. In our analysis, we took the Eulerian approach to describe the velocity field. We used Cartesian coordinate system with z-axis vertical upward and the origin is on the undisturbed water level.

The two governing Equations of tsunami hydrodynamics are the Euler’s equation of motion (Eq. 1) and the equation of continuity (Eq. 2).
\[
\frac{DV}{Dt} = K - \frac{1}{\rho} \nabla p
\]  
\(\text{Eq. 1}\)

\[
\frac{\partial p}{\partial t} = -\nabla \cdot (\rho V)
\]  
\(\text{Eq. 2}\)

The volume we consider is \(V\). Forces acting on this volume are pressure \(p\) and an external body force (per unit mass) is \(K\). For Eq. 1, it is only appropriate for large scale motion of water. For small scale motion, it is required to include the effect of viscosity, the equation of which is generally non-linear and could be solved only numerically. When the fluid is incompressible (e.g., water), the density is constant, and Eq. 2 becomes

\[
\nabla \cdot V = 0
\]  
\(\text{Eq. 3}\)

In real cases, the Euler's equation of motion and the equation of continuity should include the advection terms, the Coriolis force and bottom friction. In linear case, however, for simplicity, we could neglect these nonlinear terms. And if we assume that velocity field is uniform in \(y\) direction, the \(x\) and \(z\) components of velocity \(V\) are \((u, w)\). We could write the equations as follows:

\[
\frac{Du}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x}
\]  
\(\text{Eq. 4}\)

\[
\frac{Dw}{Dt} = -g - \frac{1}{\rho} \frac{\partial p}{\partial z}
\]  
\(\text{Eq. 5}\)

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0
\]  
\(\text{Eq. 6}\)

The boundary conditions are:

\[
w(x, z, t)\big|_{z=-d} = 0
\]  
\(\text{Eq. 7}\)

\[
p(z)\big|_{z=h} = p_0
\]  
\(\text{Eq. 8}\)

\[
w(x, z, t)\big|_{z=h} = \frac{\partial h}{\partial t}
\]  
\(\text{Eq. 9}\)

We assume \(w(x, z, t) = w(z)\sin(kx - \omega t)\) as the analytical solution, so we could calculate the phase velocity

\[
c = \frac{\omega}{k} = \sqrt{\frac{g\lambda}{2\pi}} \tanh\left(\frac{2\pi d}{\lambda}\right) \approx \sqrt{gd}
\]  
\(\text{Eq. 10}\)

When the horizontal scale of motion, or the wavelength, is much larger than the water depth, the vertical acceleration of water is negligible compared to gravity. This means that the horizontal motion of water mass is almost uniform from the ocean bottom to the surface. Such a wave is called a shallow-water wave or a long wave. Therefore, the phase velocity is only determined by the water depth. In this case, the source area of Haida Gwaii Earthquake is about 100 km, the depth of Pacific Ocean on average is about 5 km. So the long-wave approximation is appropriate and the phase velocity could be calculated as around 800 km/h.

The numerical computations on actual bathymetry have been popularly made. We took the staggered (leap-frog) grid system for average velocity and water height. The program took the input initial condition (surface displacement) once the earthquake occurred, then it computed the velocity and water height grid by grid. Figure 2 is a numerical scheme presented for a one-dimensional finite-difference computation. [1]

![Figure 2: A numerical scheme presented for a one-dimensional finite-difference computation. (Satake 2015)](image)

Error would increase as time increases unless the Courant-Friedrichs-Lewy (CFL) condition in two-dimensional is met:
\[ \Delta t \leq \frac{\Delta x}{\sqrt{2gd}} \]  

(11)

In our simulation, the time interval \( \Delta t = 5 \text{ s} \), and the spatial grid size \( \Delta x = 5 \text{ arc-minutes} \), so CFL condition could be met. In addition, when it comes to boundary condition, land-ocean boundary was adopted. The simplest assumption was a total reflection of energy on the coast.

4. Results of simulation

Figure 3 demonstrates the simulated propagation of the tsunami generated by 2012 Haida Gwaii Earthquake. The simulation was run up to 24 hours after earthquake and some snapshots were taken.

The tsunami generated on the west coast of British Columbia, Canada. It arrived in Alaska at 2 hours after earthquake. It did propagate across the Pacific Ocean but the amplitude decreased as the distance became further. It arrived in Hawaii at 5 hours 30 minutes after earthquake, and arrived in Kamchatka Peninsula at 7 hours after earthquake. It reached Japan at around 9 hours after earthquake. However, the influence was limited mainly because of the long distance for tsunami propagation and relatively lower magnitude of the earthquake. In addition, Figure 4 shows the maximum height of tsunami propagation. It is clear that the region where the maximum exceeds 0.1 m was only limited to the west coast of Canada.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Fig3.png}
\caption{The simulated propagation of tsunami after 2012 Haida Gwaii Earthquake. The tsunami propagated across the Pacific Ocean but had a limited influence on Japan.}
\end{figure}
Relatively lower water height could explain why the damage caused by tsunami was not very serious.

Having obtained simulated results, we would compare them with observed data. Many methods are utilized to observe tsunami, such as tide gauges at harbors, GPS wave gauge, and deep-ocean measurements.

Deep-ocean measurements of tsunamis have been made by using bottom pressure gauges for the early detection and warnings of a tsunami. The Deep-Ocean Assessment and Report of Tsunami (DART) records water levels using bottom pressure gauges and sends signals to a surface buoy via acoustic telemetry in the ocean, and then via satellites to a land station in real time. [1]

**Figure 4:** The maximum height of tsunami propagation. The color bar ranging from white, yellow to red, black represents different heights.

**Figure 5:** The distribution of DART stations (red triangles) in the region where tsunami would propagate. 4 DART stations whose observed waveforms would be compared with simulated waveforms are marked in the circle. The fault mechanism is also plotted.

**Figure 6:** Comparison between observed waveforms and simulated waveforms. Red lines represent observed waveforms recorded by DART stations while blue lines represent simulated waveforms.

In Figure 5, red triangles represent the locations of DART observations in the region where tsunami would propagate. These DART records waveforms after earthquake and could
be compared with simulated waveforms in the same place. Figure 6 shows the comparison of waveforms of DART03, 09, 10 and 19 and their locations could be seen clearly in Figure 5. Red lines represent observed waveforms by DART stations while blue lines represent simulated waveforms.

We could find that simulated waveforms could fit well with observed waveforms to some extent. In the beginning, there existed fluctuation in red lines before the main peak arrived, which may result from the earthquake. As time passed by, the difference between simulated waveforms and observed waveforms turned out to be larger, which may be attributed to errors in determining the parameters of single-fault model, errors in calculation of seafloor displacement, as well as other kinds like errors in staggered grid system. We could find that the simulated result would arrive a bit earlier than the observed result, and the further DART station was, the larger the difference would be. Shingo Watada et al. examined the phenomenon of tsunami delay in 2014. For simplicity, we used linear cases in numerical computation, but nonlinear effects were not the main cause of the tsunami delay recorded by the DART systems, because the travel-time delay was observed in deep oceans where nonlinear effects are negligible. [5]

Some reasons could explain this phenomenon. The tsunami travel distance on the elliptic Earth is different from that on a spherical Earth. Reference gravity is smaller in the equatorial regions of the elliptic Earth than in the spherical Earth. [5] However, in our calculation, we assumed the gravitational acceleration was a constant. Besides, the effects of the elasticity of the sea bottom and the compressibility of seawater on tsunami speed could also result in such delay. [6]

5. Improvement of the Method
From the results shown above, it is clear that though our simulated waveforms had some consistency with observed data, the error still existed. In order to reduce the error, we should improve our method of tsunami simulation. Kenji Satake has mentioned that the use of detailed bathymetry data with a small grid size is more effective than to include nonlinear terms in tsunami computation in order to reduce the difference between simulated waveforms and observed waveforms. [7] In our simulation, however, the grid size was limited to 5 arc-minutes as we used Globe.5m.grd in our computation. If finer global grid could be adopted, and that it still satisfied the CFL Stability Condition in two dimensions, the results would be more precise. But it is meaningless to reduce the time interval only.

![Figure 7](single-fault-to-multi-fault.png)

**Figure 7**: Improvement from single-fault model to multi-fault model.

One possible solution to improve our simulation is to use multi-fault model instead of single-fault model. Previously, we used only one single fault that was responsible for tsunami generation. In fact, we could divide it into several faults by conducting trial and error comparisons of observed and computed tsunami waveforms or coastal tsunami heights. The fault plane is divided into several subfaults, and the seafloor displacement is computed for each subfault with a unit amount of slip. [1]

Using this as an initial condition, tsunami waveforms are numerically computed via actual bathymetry. The observed tsunami waveforms could be expressed as a superposition of waveforms computed for each subfault as
where \( A_{ij}(t) \) is the computed waveform at the \( i^{th} \) station from the \( j^{th} \) subfault, \( b_i(t) \) is the observed waveform at the \( i^{th} \) station and \( x_j \) is the slip of \( j^{th} \) subfault. The slip \( x_j \) on each subfault could be estimated by a least-square inversion. [1]

Therefore, in our case, we divided our fault plane into 4 homogeneous subfaults, which had same length, width, strike, dip and rake, but different depth, location and slip. The constraint for least-square inversion was that the sum of slips of 4 subfaults should become 10.0 m in order to have the same magnitude as single-fault model. We took the least-square inversion, conducting trial and error comparisons of observed and computed tsunami waveforms in 8 DART stations during the certain period after the earthquake. Finally we obtained the parameters for 4 subfaults shown in the Table 1.

**Table 1:** Parameters for 4 subfaults obtained via least-square inversion. The sum of slips should become 10.0 m.

<table>
<thead>
<tr>
<th>Subfault</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>55.0 km</td>
<td>55.0 km</td>
<td>55.0 km</td>
<td>55.0 km</td>
</tr>
<tr>
<td>Width</td>
<td>25.0 km</td>
<td>25.0 km</td>
<td>25.0 km</td>
<td>25.0 km</td>
</tr>
<tr>
<td>Depth</td>
<td>1.0 km</td>
<td>1.0 km</td>
<td>11.6 km</td>
<td>11.6 km</td>
</tr>
<tr>
<td>Strike</td>
<td>314°</td>
<td>314°</td>
<td>314°</td>
<td>314°</td>
</tr>
<tr>
<td>Dip</td>
<td>25°</td>
<td>25°</td>
<td>25°</td>
<td>25°</td>
</tr>
<tr>
<td>Rake</td>
<td>100°</td>
<td>100°</td>
<td>100°</td>
<td>100°</td>
</tr>
<tr>
<td>Slip</td>
<td>0.75 m</td>
<td>2.05 m</td>
<td>4.33 m</td>
<td>2.87 m</td>
</tr>
<tr>
<td>Lat</td>
<td>52.1° N</td>
<td>52.4° N</td>
<td>52.3° N</td>
<td>52.6° N</td>
</tr>
<tr>
<td>Lon</td>
<td>131.7° E</td>
<td>132.3° E</td>
<td>131.5° E</td>
<td>132.0° E</td>
</tr>
</tbody>
</table>

We used the corresponding 4 subfaults to simulate tsunami generation and propagation. Then we compared the simulated waveforms with multi-fault model, simulated waveforms with single-fault model and observed waveforms together of DART stations. It could be seen from the Figure 8 that simulated waveforms with multi-fault model fit better with observed waveforms than those with single-fault model, especially in early time. As time passed by, the difference between simulated waveforms and observed waveforms still increased, either with multi-fault model or single-fault model.

**Figure 8:** Comparison between observed waveforms simulated waveforms. Red lines represent observed waveforms recorded by DART stations, blue lines represent simulated waveforms with single-fault model and green lines represent simulated waveforms with multi-fault model.

6. Further Discussion

In our further work, we could use finer grid (e.g. 1 arc-minute) in order to get more precise results. In addition, since we have already adopted multi-fault model, we could consider not only spatial slip distribution of subfaults but also temporal slip distribution of them. In other words, the subfaults do not slip simultaneously but with a time difference. Based on these assumption, we could still use least-square inversion and obtain a better simulation of tsunami generation and propagation.
Moreover, we could also use the method of tsunami simulation to analyze hypothetical earthquake happened in the certain region we care, calculated the height of generated tsunami, and assess possible hazard caused by tsunami of some countries or cities.

7. Conclusion
The 2012 Haida Gwaii earthquake occurred off the west coast of Haida Gwaii and generated tsunami that propagated across the Pacific Ocean.
At first, we used Okada’s program with parameters of single-fault model to simulate tsunami generation and the initial condition for the calculation was the initial displacement of water surface which was assumed exactly same as the seafloor displacement due to faulting.
We adopted staggered grid system to simulate tsunami propagation up to 24 hours after earthquake. In our computation, long-wave approximation was adopted. The phase velocity of tsunami was about 800 km/h. The tsunami arrived in Alaska at 2 hours after earthquake, in Hawaii at 5 hours 30 minutes after earthquake, and in Kamchatka Peninsula at 7 hours after earthquake. It reached Japan at around 9 hours after earthquake but with a very limited influence.
Then, we compared the simulated waveforms with observed waveforms recorded by DART stations. We found that simulated waveforms could have consistency with observed waveforms to some extent, and that the simulated results would arrive a bit earlier than the observed results. This phenomenon could be attributed to the real shape of elliptic Earth, the effects of the elasticity of the sea bottom and the compressibility of seawater.
Furthermore, we tried multi-fault model to describe the earthquake and to simulate tsunami generation. In the early time, simulated waveforms with multi-fault model fit better with observed waveforms than those with single-fault model. As time passed by, the difference between simulated waveforms and observed waveforms still increased, either with multi-fault model or single-fault model.

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Reference
2014.