Feasibility Study of Tensor Interaction in Leptonic Tau Decays at Belle and Belle II

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Abstract
Tensor Interaction in leptonic $\tau$ decays was last studied by the DELPHI @ LEP collaboration and in that experiment that statistical uncertainty was 3.6%. Here, I show by means of a feasibility study that the Belle and Belle II collaborations with their extremely high luminosities can improve upon the uncertainty in measuring tensor interaction substantially.

1 Introduction

The matter-antimatter asymmetry present in our universe has been the topic of active research for quite sometime now. Charge parity (CP) violation was one of the three conditions proposed by Sakharov that could explain this asymmetry. Theorized by M. Kobayashi and T. Maskawa in extension of N. Cabibbo’s work, CP violation exhibits itself as a complex phase in the quark-mixing matrix, also known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Investigating CP violation in the B-meson sector was the primary goal of the two B-collaborations Belle and BaBar. In the summer of 2001, the presence of CP violation in the B meson system was established by the Belle collaboration and simultaneously by the BaBar collaboration through the measurement of the time dependent asymmetry in the decay process $B^0(\bar{B}^0) \rightarrow J/\psi K_S^0$. The experimental data indicated that the Kobayashi-Maskawa mechanism, which is now a part of the SM of elementary particles, is indeed the dominant source of the observed CP violation in nature. Following the experimental confirmation, M. Kobayashi and T. Maskawa were awarded the 2008 Nobel Prize for physics. However, the amount of CP violation predicted by the Kobayashi-Maskawa formulation is not enough to explain the degree of existing asymmetry present and hence the need arises to test the various new theoretical models constantly being churned out in this sector. With this view, the Belle detector is being upgraded to Belle II, which aims to collect 50 times more data than Belle.
Figure 1: The Total Simulation

Such a B-factory also provides a clean environment to study leptonic \( \tau \) decays. Leptonic decays of a \( \tau \) lepton are especially well suited to investigate the structure of charged weak interactions [1]. I was interested in studying the Tensor Interaction involved in such \( \tau \) decays. The Tensor interaction manifests itself by modifying the leptonic energy spectrum. I developed a generator in ROOT to simulate the total reaction as shown in Fig.1 using Monte-Carlo techniques. I then developed a fitter to extract the sensitivity of the Belle and Belle II detectors towards Tensor Coupling, which was the primary goal of my study.

2 Theory

Before we go into the methodology of developing the simulator and the fitter, let us have a brief look at the various theoretical considerations involved in my feasibility study.

2.1 Tensor Interaction

First, let us concentrate on the leptonic decay of the \( \tau^- \) and see how the leptonic energy spectrum is actually modified by different degrees of Tensor Coupling. Tensor interaction modifies the vertex function, see Fig.2 by introducing an additional dependence on the momentum of the \( W^- \) boson via the term \( \frac{\alpha_\tau}{2M_\tau} \sigma^{\mu\nu} q_\nu \) where \( q_\nu \) is the momentum of the \( W^- \) boson, \( M_\tau \) is the \( \tau \) rest mass, \( \sigma^{\mu\nu} = \frac{i}{2} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) \) \((\gamma^{\mu}, \mu = 0 \div 3 \text{ are Dirac matrices})\) and \( \alpha_\tau \) is dimensionless tensor coupling constant. The differential decay probability to obtain a lepton with reduced energy \( x = \frac{E_\ell}{E_{\text{max}}} \) emitted at a particular solid angle \( \Omega \) is given by[1]
\[ \frac{d^2\Gamma}{dx d\Omega} = c_1 \sqrt{x^2 - x_0^2} \left[ \frac{x^2}{3} + \frac{x}{2} - \frac{x_0^2}{6} + \frac{x^2}{6} \alpha_\tau \pm \frac{1}{3} P_\tau \cos \theta \left( \sqrt{x^2 - x_0^2} \left( x - 1 + \frac{1 - x_0^2}{2} \right) + x^2 \alpha_\tau \right) \right] \] (1)

where \( c_1 \) is a constant, \( x_0 = \frac{m_\ell}{E_{\text{max}}} \), \( E_{\text{max}} = \frac{M_L}{2} \left( 1 + \frac{m_\ell^2}{M_L^2} \right) \), \( P_\tau \) is the \( \tau \) polarization and \( \theta \) is the angle between \( \tau \) spin and \( \vec{p}_\ell \), i.e. momentum of lepton. I would just like to point out that this decay width is in the \( \tau \) rest frame.

We simulated this \( \tau \) decay using different values of \( \alpha_\tau \) and obtained energy spectra are shown in Fig. 3. As is evident from Fig. 3 higher values of \( x \) result in more prominent separation between the distributions corresponding to different values of \( \alpha_\tau \). Zoomed high energy part of the spectrum is shown in Fig. 4.

### 2.2 Decay Rates with Measured Variables

As illustrated in the previous section, the leptonic energy spectrum is distorted substantially by the Tensor Interaction and this provides a good opportunity of measuring \( \alpha_\tau \). However, at Belle and Belle II we cannot uniquely determine the \( \tau^- \) direction. We can only limit the \( \tau \) direction within two angles \( \phi_1 \) and \( \phi_2 \) using the missing mass constraints introduced by the two decays \( \tau^+ \to \rho^+ \nu_\tau \) and \( \tau^- \to \ell^- \nu_\ell \nu_\tau \). I will not go into the details of how to calculate \( \phi_1 \) and \( \phi_2 \) due to the length of the calculation. An interested reader may refer to Ref.[1]. A brief glimpse of the calculation scheme as well the expressions of \( \phi_1 \) and \( \phi_2 \) are shown in Fig.5. The differential cross section in terms of measured variables is actually obtained as an integration over \( \phi \) from \( \phi_1 \) to \( \phi_2 \) as shown below.\(^1\)

\(^1\)In Eq.2 variables with asterisk refers to the \( \tau \) rest frame and variables with tilde refers to the \( \rho^+ \) rest frame. Other variables are in the CMS frame. This convention is followed throughout the rest of the article and any exceptions shall be pointed out.
Figure 3: Lepton energy spectrum for different values of tensor coupling, $\alpha$.

$$\frac{d\sigma(\ell^-,\rho^+)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_\pi^2 d\Omega_\tau} = \int_{\Phi_1}^{\Phi_2} F_\sigma^* (\ell^-,\rho^+) \left| \frac{\partial (E_\ell^* \Omega_\ell^* \Omega_\rho^* \Omega_\tau)}{\partial (p_\ell \Omega_\ell p_\rho \Omega_\rho \Phi_\tau)} \right| d\Phi_\tau$$ (2)

where the Jacobian can be reduced to a product of two degraded Jacobians given by

$$\left| \frac{\partial (E_\ell^* \Omega_\ell^* \Omega_\rho^* \Omega_\tau)}{\partial (p_\ell \Omega_\ell p_\rho \Omega_\rho \Phi_\tau)} \right| = \left| \frac{\partial (E_\rho^* \Omega_\ell \Omega_\rho \Phi_\tau)}{\partial (p_\rho \Omega_\rho \Phi_\tau)} \right| \left| \frac{\partial (\Omega_\rho^* \Omega_\tau)}{\partial (p_\rho \Omega_\rho \Phi_\tau)} \right|$$ (3)

The first Jacobian originates in the leptonic decay $\tau^- \rightarrow \ell^- \nu_\ell \tau$ and the second one in the semileptonic decay $\tau^+ \rightarrow \rho^+ \nu_\tau$. These two Jacobians are given by

$$\left| \frac{\partial (E_\ell^* \Omega_\ell^*)}{\partial (p_\ell \Omega_\ell)} \right| = p_\ell \left\{ \left( \frac{p_\ell}{E_\ell p_\ell^*} \right) \left( \frac{cos^2 \phi_\ell^*}{A_\ell^2} \right) sin^2 \theta_\ell \right\}$$ (4)

$$\left| \frac{\partial (\Omega_\rho^* \Omega_\tau)}{\partial (p_\rho \Omega_\rho \Phi_\tau)} \right| = M_\tau \left\{ \left( \frac{p_\rho}{E_\rho p\rho^*} \right) \left( \frac{cos^2 \phi_\rho^*}{A_\rho^2} \right) sin^2 \Theta \right\}$$ (5)

$$A_i = cos(\phi_i - \phi_\tau)sin\theta_i cos\theta_\tau - cos\theta_i sin\theta_\tau$$ (6)

In equations 3-5 $i = \ell$ or $\rho$, $cos\theta_\tau = \vec{\eta}_\tau^2$, $p_\rho^* = (M_\tau^2 - m_\rho^2) / 2M_\tau$ and all other symbols have their usual significance.

$^2\vec{\eta}$ is a unit vector along the momentum of the corresponding particle.
Figure 4: Zoomed high energy part of the lepton energy spectrum for different values of tensor coupling, $\alpha$.

In the $\ell - \rho$ events the direction of $\tau$ axis is constrained by arc, which is determined by measurable angles:

$$\phi_1 = \pi + \arcsin \left( \frac{\cos \psi \cos \alpha + \cos \chi}{\sin \psi \sin \alpha} \right), \quad \phi_2 = 2\pi - \arcsin \left( \frac{\cos \psi \cos \alpha + \cos \chi}{\sin \psi \sin \alpha} \right)$$

Figure 5: Calculation scheme of $\phi_1$ and $\phi_2$
Now let us see the expression for the $F^*_{\sigma} (\ell^-, \rho^+)$ as mentioned in Eq.2.

$$F^*_{\sigma} (\ell^-, \rho^+) = \frac{d\sigma}{d\Omega_\ell dx^* d\Omega_\rho^* dm_{\pi^0}} =$$

$$\kappa [D_0 A^* A^* + (D_{ij} \eta_{ij}^* B_j^* B^*)] \text{BPS}$$

(7)

where $\kappa$ is a constant and

$$D_0 = 1 + \cos^2 \theta_\tau + \frac{1}{\gamma^2} \sin^2 \theta_\tau$$

(8)

$$D_{ij} = \begin{pmatrix}
(1 + \frac{1}{\gamma^2}) | \sin^2 \theta_\tau & 0 & \frac{\sin 2\theta_\tau}{\gamma} \\
0 & -\beta^2 \sin^2 \theta_\tau & 0 \\
\frac{\sin 2\theta_\tau}{\gamma} & 0 & 1 + \cos^2 \theta_\tau - \frac{\sin^2 \theta_\tau}{\gamma}
\end{pmatrix}$$

(9)

$D_0$ and $D_{ij}$ arise from the $\tau$ pair production and all symbols have their usual significance.

$$A(x^*) = \sqrt{x^2 - x_0^2} \left[ -\frac{x^2}{3} + \frac{x^2}{2} - \frac{x_0^2}{6} + \frac{1}{3} x^2 \alpha_\tau \right]$$

(10)

$$B(x^*) = \frac{1}{3} \sqrt{x^2 - x_0^2} P_2 \left[ \sqrt{x^2 - x_0^2} \left( x - 1 + \frac{1 - x_0^2}{2} \right) + x^2 \alpha_\tau \right]$$

(11)

$A^*$ and $B^*$ arise from the $\tau^-$ decay where both $x$ and $x_0$ correspond to the explanations given earlier.

$$A^* = 2 (w_1^* - w_2^*) P_2 Q - E_2^* Q^2$$

(12)

$$B_j^* = B_{1j}^* q_{1j}^* + B_{2j}^* q_{2j}^*$$

(13)

$A^{*\prime}$ and $B_j^{*\prime}$ arise from the $\tau^+$ decay and in Equations 12 and 13, $P_1 = (E_1^*, \rho_1^*)$, $P_2 = (E_2^*, \rho_2^*)$, $q_1 = (\omega_1^*, \bar{q}_1^*)$, $q_2 = (\omega_2^*, \bar{q}_2^*)$ correspond to the four vector momentum of the $\tau^+, \nu_\tau, \pi^+$ and $\pi^0$ particles respectively in the Tau Rest Frame. Besides, $Q = q_1 - q_2$.

Finally we have to determine the BPS mentioned in Eq.7.

$$\text{BPS} = \left| BW \left( m^2_{\pi\pi} \right) \right|^2 \frac{2 \left| \rho_2^* \right|}{M_\tau} \frac{2 \left| \rho_\tau \right|}{m_{\pi\pi}}$$

(14)

where
\[
\text{BW} \left( m_{\pi\pi}^2 \right) = \frac{m_{\rho}^2}{m_{\rho}^2 - m_{\pi\pi}^2 - im_{\rho} \Gamma_{\rho} \left( m_{\pi\pi}^2 \right)}
\]

(15)

\[
\Gamma_{\rho} \left( m_{\pi\pi}^2 \right) = \Gamma_{\rho_0} \frac{m_{\rho}}{m_{\pi\pi}^2} \left( \frac{\left| \vec{p}_\pi \right| (m_{\pi\pi}^2)}{\left| \vec{p}_\pi \right| (m_{\rho}^2)} \right)
\]

(16)

In equations 14 - 16 \( m_{\pi\pi} \) refers to the invariant mass of \( \pi\pi^0 \) system, \( \left| \vec{p}_\pi \right| (m_{\pi\pi}^2) \) and \( \left| \vec{p}_\pi \right| (m_{\rho}^2) \) refer to the pion momenta in the \( \rho^+ \) rest frame with the mass of \( \rho^+ \) taken as \( m_{\pi\pi} \) and \( m_{\rho} \) respectively.

### 3 The Simulation

Using the differential cross section given in Eq.2 and general Monte-Carlo simulation techniques, I wrote down a generator in ROOT to simulate the electron-positron collision. Thereafter, I created numerous data sets each corresponding to a million events and a different value of the tensor coupling \( \alpha_{\tau} \). The values of the various other parameters were taken from [3]. After this, I developed a fitter which extracted the values of alpha from these data sets. The error of these fits, after proper scaling to the number of events at Belle and Belle II, gave me the sensitivity of the detectors towards Tensor Coupling.

#### 3.1 The Fitter

**The Likelihood Method** I developed special program (fitter) to perform unbinned maximum likelihood fit of generated events. The maximum likelihood method is actually a pre-cursor to the popular \( \chi^2 \) method where we do away with the assumption that the uncertainty can be interpreted as a Gaussian. This method stands on minimizing the likelihood which is basically the probability density function evaluated at a particular data point. For measurements consisting of an ensemble of data points, the likelihood is simply the product of each observation as shown below

\[
L(p) = \prod_{i=1,...,N} f(x_i, p)
\]

(17)

where \( p \) corresponds to the parameter of interest and \( N \) refers to the total number of events. The predicted value of \( p \) is that value which corresponds to the minimum value of the negative log likelihood function given by

\[
\mathcal{L} = -logL(p) = - \sum_{i=1,...,N} log f(x_i, p)
\]

(18)
Estimating the statistical uncertainty using the likelihood

The simplest way to measure the statistical uncertainty \( \sigma(\hat{p}) \) on the estimate of a single parameter \( \hat{p} \) is to estimate the variance \( V(\hat{p}) \) of that parameter and calculate the uncertainty as the square-root of the variance. The ML estimator for the variance on \( bp \) is given by the second derivative of the log-likelihood at \( p = \hat{p} \):

\[
\sigma(\hat{p})^2 = V(\hat{p}) = \left( \frac{d^2 L}{d^2 p} \right)_{p=\hat{p}}^{-1} \tag{19}
\]

The exact method to get the uncertainty of a parameter is based on an interval defined by the log-likelihood ratio

\[
\lambda(p) = \log \frac{L(p)}{L(\hat{p})} \tag{20}
\]

where \( L(p) \) is the likelihood for a given value \( p \), \( \hat{p} \) is the value of \( p \) for which the likelihood is maximal and \( L(\hat{p}) \) is therefore the maximum value of the likelihood. An interval in \( p \) defined by a rise in the log-likelihood-ratio of half a unit from zero corresponds to nominally a 68% confidence interval.

Our Case

In our case, the probability density function is given by

\[
f(\alpha_\tau) = \frac{K_0 + \alpha_\tau K_1}{C_0 + \alpha_\tau C_1}
\]

where the numerator is just a reworking of the differential cross section given by Eq.2 and \( C_0, C_1 \) are the normalization coefficients corresponding to \( K_0 \) and \( K_1 \). Therefore the negative log likelihood function is given by

\[
\mathcal{L} = -\log(L) = -\sum_{i=1}^{N} \log \left[ \frac{K_0^i + \alpha_\tau K_1^i}{C_0^i + \alpha_\tau C_1^i} \right] \tag{21}
\]

The robustness of the fitter is proved by Fig.6 which shows the linearity between the predicted and the actual values.

3.2 The Number of Events

Here, I show how the total number of selected events at Belle were calculated and how we scaled our result to match the huge amount of statistics available at Belle and expected at Belle II.

The integrated luminosity and the cross section corresponding to the \( \tau \) pair production at Belle are given by

\[
L = \int \mathcal{L} dt = 1ab^{-1} = 10^9 nb^{-1} \tag{22}
\]

\[
\sigma_{\text{tot}} [e^+e^- \to \tau^+\tau^-] (E = 10.58GeV) = (0.919 \pm 0.002) nb \tag{23}
\]
Figure 6: Predicted Values v/s actual values of $\alpha_\tau$

Hence, the total number of events, $N_{tot} = \mathcal{L} \sigma_{tot} = 0.92 \times 10^9$. To obtain the number of events corresponding to the decay mode we are interested in this number must be multiplied by the appropriate branching fractions and detection efficiency, $\varepsilon_{det}$:

$$N_{sel} = 2 \times N_{tot} Br(\tau^\pm \rightarrow \rho^\pm \nu) \left[ Br(\tau^\pm \rightarrow e^\pm \nu \bar{\nu}^+ ) + Br(\tau^\pm \rightarrow \mu^\pm \nu \bar{\nu}^+ ) \right] \times \varepsilon_{det}$$

$$= 2 \times 0.092 \times 10^9 \times 0.255 \times (0.174 + 0.178) \times 0.012 = 19.8 \times 10^6$$

(24)

Therefore the number of selected events at Belle are close to 20 million and the plan is to collect 50 times more data at Belle II. We had simulated one million events. Therefore, we scale the errors corresponding to $\alpha_\tau$ using the following formulae:

$$\frac{\delta_{sim}(\alpha_\tau)}{\sqrt{19.8}} = \delta_{Belle}(\alpha_\tau)$$

(25)

$$\frac{\delta_{Belle}(\alpha_\tau)}{\sqrt{50}} = \delta_{BelleII}(\alpha_\tau)$$

(26)

4 Results and Discussion

Tensor interaction was also studied by the DELPHI @ LEP (CERN) collaboration [4]. Their results along with my own results are presented in Table.1.
Table 1: Summary of the statistical uncertainties of tensor coupling $\alpha_\tau$.

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<tr>
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<th>DELPHI</th>
<th>Belle</th>
<th>Belle II</th>
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<tr>
<td></td>
<td>3.6%</td>
<td>0.0506%</td>
<td>0.0072%</td>
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However, we must remember the fact, that these are only statistical uncertainties and the measurement of $\alpha_\tau$ at Belle and Belle II will be dominated by systematic uncertainties. At Belle and Belle II it is possible to limit the systematic uncertainties to 0.3% - 1.0% and at DELPHI this was 1.8%.[4]

Therefore at Belle/Belle II, we can expect an improvement in the measurement of Tensor Coupling by a factor of 5 to 15. Hence, it is strongly recommended that Tensor Interaction be studied currently with Belle data and in the future with Belle II data, when it becomes available.

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References


